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WHAT IS THE “SIZE” OF INFINITY?

What is the “size” of infinity? In set theory, the “size” of a set refers to the number of elements in the set known as the cardinality of the set. However, infinity is not a set, so the “size” is irrelevant to cardinality for a set. Just for the sake of argument, the “size” (borrowed from cardinality) is to ask how many “infinities” are out there. First, are aleph numbers the infinity? No. We have to rule out aleph numbers being infinity. An aleph number is an infinite number, which is a number, but infinity is not a number, so the aleph numbers are not infinity.

Prove that each aleph number is not infinity.

Proof: Let $[\![\,]\!]=\{\aleph_0, \aleph_1, \aleph_2, \aleph_3, ..., \aleph_\aleph\}$. Let C denote a container.

Suppose $\aleph_0$ is ∞ $\therefore$ $\aleph_0$ in ∞ $\therefore$ ∞ is C $\therefore\infty$ belongs to C $\therefore\infty$ is bounded, by Huo Jian Hua’s Definition of Boundedness. But, ∞ is not bounded, (∞ is not bounded) and (∞ is bounded) $\rightarrow\leftarrow\forall\aleph_0$ $\therefore\aleph_0$ is not ∞.

Suppose $\aleph_1$ is ∞ $\therefore$ $\aleph_1$ in ∞ $\therefore$ ∞ is C $\therefore\infty$ belongs to C $\therefore\infty$ is bounded, by Huo Jian Hua’s Definition of Boundedness. But, ∞ is not bounded, (∞ is not bounded) and (∞ is bounded) $\rightarrow\leftarrow\forall\aleph_1$ $\therefore\aleph_1$ is not ∞.

Assume $\aleph_n$ is not ∞. Suppose $\aleph_{n+1}$ is ∞ $\therefore$ $\aleph_{n+1}$ in ∞ $\therefore$ ∞ is C $\therefore\infty$ belongs to C $\therefore\infty$ is bounded, by Huo Jian Hua’s Definition of Boundedness. But, ∞ is not bounded, (∞ is not bounded) and (∞ is bounded) $\rightarrow\leftarrow\forall\aleph_{n+1}$ $\therefore\aleph_{n+1}$ is not ∞. $\therefore\aleph_n$ is not ∞ $\forall\aleph_n\in[\![\,]\!]$. Q.E.D.

Prove that any aleph number is not infinity.

Proof: Let x be any aleph number. Suppose x is infinity $\therefore$infinity is a container $\therefore$infinity belongs to a container $\therefore$infinity is bounded, <u>by Huo Jian Hua’s Definition of Boundedness</u>. But, infinity is not bounded; (infinity is not bounded) and (infinity is bounded) $\rightarrow\leftarrow\forall$infinity, x, $\therefore$x is not infinity $\forall$x $\therefore$any aleph number is not infinity. Q.E.D.

Further, from “A Summary of the Unbounded” section, unbounded is not infinity although infinity is unbounded by axiom, so the caveat is this: even if each aleph number may be unbounded, the aleph numbers are not infinity.

We have proved that each and any aleph number is not infinity. Further, the aleph numbers are just numbers that have their own rules that cannot be applied to infinity. For example, $\aleph_0=\aleph_0$, but $\infty\neq\infty$ $\forall\infty$ for “infinity is unbounded” axiom to hold. Also, $0 < \aleph_0 \leq \aleph_1 \leq \aleph_2 ... < +\infty$, so the $\aleph_0$ is bounded from below by 0 and from above by $\aleph_1$, and $\aleph_1$ is bounded from below by $\aleph_0$ and from above by $\aleph_2$ ..., etc, so therefore each aleph number is bounded, but infinity is unbounded.

Prove that $\aleph_0$ is reflexive.

Proof:

$1=1$ $\forall 1\in\mathbb{N}$. Assume n=n, so $n+1=n+1 \Rightarrow n=n$ $\therefore$n is reflexive $\forall n\in\mathbb{N}$. $\aleph=\aleph_0$, also $\aleph\in\mathbb{N}$. $\therefore$ $\aleph$ is reflexive. $\therefore\aleph_0$ is reflexive. Q.E.D.

Prove 2^n is reflexive $\forall n\in\mathbb{N}$.

Proof:

$2^1 = 2$, $2=2$ $\therefore$ 2^1 is reflexive. Assume 2^n is reflexive. $2^{n+1} = (2)2^n$ $\therefore$ $(2)2^n = (2)2^n$ $\therefore 2^n=2^n$ $\forall n\in\mathbb{N}$. Q.E.D.

Prove $\aleph_n$ is reflexive $\forall n \in \mathbb{N}$.

Proof: $\aleph_1 = 2^{\aleph}$, $\aleph \in \mathbb{N}$ ∴ $2^{\aleph}$ is reflexive ∴$\aleph_1$ is reflexive .
Assume $\aleph_n$ is reflexive. $\aleph_n = 2^{\aleph_{n-1}}$ ∴ $2^{\aleph_{n-1}} = 2^{\aleph_{n-1}}$.
$\aleph_{n+1} = 2^{\aleph_n} = 2^{(2^{\aleph_{n-1}})}$ ∴ $2^{\aleph_n} = 2^{(2^{\aleph_{n-1}})}$ ∴ $\ln(2^{\aleph_n}) = \ln\left(2^{(2^{\aleph_{n-1}})}\right) \Rightarrow \aleph_n \ln(2) = (2^{\aleph_{n-1}})\ln(2)$.
But, $\aleph_n = 2^{\aleph_{n-1}}$ ∴ $\aleph_n \ln(2) = (\aleph_n)\ln(2) \Rightarrow \aleph_n = \aleph_n$ ∴$\aleph_n$ is reflexive $\forall n \in \mathbb{N}$. Q.E.D.

Side Discussion: Let k denote an infinite whole number such that $0<k<\aleph$ ∴$k \in \mathbb{N}$. ∵2^n is reflexive $\forall n \in \mathbb{N}$ ∴ 2^k is reflexive.

Prove $\exists \aleph_n \forall n \in \mathbb{N}$.

Proof: Each aleph number is not infinity ⟦⟧={$\aleph_0$, $\aleph_1$, $\aleph_2$, $\aleph_{3, \ldots,}$ $\aleph_{\aleph}$}. ∴$\aleph_0$ is not infinity ∴$\exists \aleph_0$; $\aleph_1$ is not infinity ∴$\exists \aleph_1$. Assume $\exists \aleph_n$. Suppose $\aleph_{n+1}$ is infinity ∴infinity is a container ∴infinity belongs to a container ∴infinity is bounded. But, infinity is not bounded, (infinity is not bounded) and (infinity is bounded) ➔🡐 $\forall \aleph_{n+1}$ ∴$\aleph_{n+1}$ is not infinity ∴$\exists \aleph_{n+1}$ ∴$\exists \aleph_n \forall n \in \mathbb{N}$. Q.E.D.

The followings summarize some distinctions between aleph numbers and infinity:

Aleph numbers are numbers.	Infinity is not a number.
Aleph numbers can exist while being reflexive.	Infinity must be irreflexive in order to exist.
Aleph numbers are bounded.	Infinity is unbounded.

Natural numbers are numbers, and each natural number is reflexive and bounded; real numbers are numbers, and each real number is reflexive and bounded; aleph numbers are numbers, and each aleph number can be reflexive and bounded. Although a natural number is not infinity, we don't call the natural number an infinity; although a real number is not infinity, we don't call the real number an infinity. The same should apply to the aleph numbers. Although an aleph number is not infinity, we should not call the aleph number an infinity. Although infinity is not infinity, it does not imply that any x being not infinity is infinity.

Is the size of infinity being infinity? No. The size of infinity is not infinity.

Prove that the size of infinity is not infinity.

Proof: Let x denote the size of infinity.

Suppose x is infinity ∴x is in infinity ∴infinity is a container of x ∴infinity is a container ∴infinity belongs to a container ∴infinity is bounded, by Huo Jian Hua's Definition of Boundedness. But, infinity is not bounded, (infinity is not bounded) and (infinity is bounded) ➔🡐∀x ∴x is not infinity ∴the size of infinity is not infinity. Q.E.D.

Does the size of infinity exist? Yes. ∃(size of infinity) ∀(size of infinity).

Prove ∃(size of infinity) ∀(size of infinity).
Proof: ∵(size of infinity) is not infinity
∴∃(size of infinity)∀(size of infinity), by Definition of Existence. Q.E.D.

We have thrown out the size of infinity being infinity, and we know that the aleph numbers are not infinity, so aleph numbers are not accounted for the size of infinity. Let us look at what an analogy says:

$\lim_{x \to 0^+} \frac{n}{x} = +\infty\ \forall n \in \mathbb{N},\ x \in \mathbb{R}$. From the analogy, there are ℵ count of ($\lim_{x \to 0^+} \frac{n}{x}$) that all map to "one" $+\infty$, so from the analogy, there are at least ℵ counts of the "one" $+\infty$!

"Size" of Infinity

So, what is the "size" of infinity? We know that infinity is not empty, so the count of infinity must not be 0, and since the "size" is associated with cardinality concept, so the "size" of infinity must not be negative because a cardinal number is not negative. From the analogy we can find at least 1 count of analogy being $+\infty$, e.g. $\lim_{x \to 0^+} \frac{1}{x} = +\infty\ \forall 1 \in \mathbb{N},\ x \in \mathbb{R}$, and therefore, based on the analogy, the "size" might be greater or equals to 1. However, because ∞ is not empty, so the count of infinity cannot be 0 ∴ "size">0, so the "size" of infinity is greater than 0, bounded from below, and thus, the "size" is bounded. All rights-benefits go to Huo Jian Hua by the guaranteed terms of use on Huo Jian Hua's Definition of Boundedness as always, and this concludes the part of discussion in this section.