OFF THE PAPER – RESOLVING AN INFINITY PARADOX

This section discusses the indeterminate existence of infinity when infinity is infinity, and thus it presents a paradox to resolve because of the IINI∀I theorem. “Infinity is not infinity” or “infinity is infinity”, which one is correct? Although mathematics does not usually prove an established axiom, confronted by the paradox we must prove that the “infinity is unbounded” axiom is false in this Off the Paper section outside of the “infinity is unbounded” axiomatics system to further resolve the paradox.

Prove that infinity is infinity ∀infinity.

Proof: By consensus, guarantees, and supports to the “infinity is unbounded” axiom, recall from the [Table A1]

infinity	unbounded	infinity is unbounded
T	T	T

[Table A1]

While the “infinity is unbounded” axiom holds,

infinity	infinity	infinity is infinity
T	T	T

[Table A1.2]

∴infinity is infinity ∀infinity. Q.E.D.

Indeterminate Existence of Infinity When Infinity is Infinity.

We have proved “infinity is infinity” while the “infinity is unbounded” axiom holds. Recall from the [Table E_N]:

col / row	1	2	3	4	5	6	7	
1	x	∃x	x is not ∞	[∃x]⇔[x is not ∞]	∄x	x is ∅	[∄x]⇔[x is ∅]	
2	T	T	T	T	F	F	T	
3	F	T	T	T	F	F	T	
4	∅	T	T	T	T	T	T	
5	not ∞	T	T	T	F	F	T	
6	∞	T	T	T	F	F	T	[Table E_N]

The Definition of Existence is in column 4, and the Definition of Nonexistence is in column 7. The “infinity is infinity” situation is neither listed from the above Definition of Existence, nor from the Definition of Nonexistence because infinity is not empty ∅. Therefore, when infinity is infinity, the existence of infinity cannot be determined by the definitions, and thus the existence of such infinity is indeterminate. Isn’t it why the existence of such infinity can be debated over ages and ages?

Now can we jump to the conclusion that indeterminate is infinity? No. Indeterminate is not infinity, and because indeterminate is not infinity, so the indeterminateness can exist in this paper for a discussion.

Prove that indeterminate is not infinity ∀indeterminate.

Proof: Suppose indeterminate is infinity ∴ infinity is a container. But, infinity is not a container; (infinity is not a container) and (infinity is a container), ➔🡐∀indeterminate, ∴indeterminate is not infinity ∀indeterminate. Q.E.D.

Prove ∃indeterminate∀indeterminate.

Proof: Indeterminate is not ∞ ∀indeterminate∴ ∃indeterminate. Restate the Definition of Existence:
∃x if and only if x is not ∞ ∀x, and equivalently, x is not ∞ if and only if ∃x ∀x.
∴ ∃indeterminate if and only if indeterminate is not ∞ ∀indeterminate,
∴ indeterminate is not ∞ if and only if ∃indeterminate ∀indeterminate. Q.E.D.

Resolving the Infinity Paradox

We have reached two opposite conclusions for infinity: [infinity is infinity ∀infinity] and [infinity is not infinity ∀infinity]. Which one is correct? Although mathematics does not usually prove an established axiom, given that we have the paradox at hand, we must prove that the "infinity is unbounded" axiom is false to further resolve the paradox.

Prove that "infinity is unbounded" axiom is false.

Proof: The approach in this proof is to show that the infinity in A="infinity is unbounded" is empty. The word "infinity" is a placeholder for ∅ in A. Because this ∅ is in A, so therefore ∅ being unbounded is a contradiction which negates A from being true.

The word "infinity" is in the axiom A, so therefore infinity belongs to A because [x in y]⇔[x belongs to y] ∀x, y.

Prove that [x belongs to y]⇔[x in y] ∀x, y.
Proof: Recall the definition of boundedness [Table D1]. Attach the [x in y], and, [x belongs to y]⇔[x in y] columns in the following:

x	y	x belongs to y	x is bounded	[x is bounded]⇔[x belongs to y]		x in y	[x belongs to y]⇔[x in y]	[x in y]⇔[x belongs to y]
T	T	T	T	T		T	T	T
T	F	F	F	T		F	T	T
T	∅	F	F	T		F	T	T
F	T	F	F	T		F	T	T
F	F	T	T	T		T	T	T
F	∅	F	F	T		F	T	T
∅	T	T	T	T		T	T	T
∅	F	T	T	T		T	T	T
∅	∅	T	T	T		T	T	T

[Table D2]

For infinity belongs to A, so therefore infinity is bounded, and the consequence is that infinity is not in A as shown in the following proof.

Prove that infinity is not in the axiom A="infinity is unbounded" ∀A, infinity.
Proof: Case 1: infinity is not in A. This proof case is done.
Case 2: suppose infinity is in A="infinity is unbounded",
∴ [infinity is in A="infinity is unbounded"] ⇒[infinity in A="infinity is unbounded"]
∴ [infinity belongs to A="infinity is unbounded"]⇒[(infinity) belongs to (A="infinity is unbounded")]⇒[infinity is bounded], by Huo Jian Hua's Definition of Boundedness.
But, infinity is not bounded, (infinity is not bounded) and (infinity is bounded), ➔🡐 ∀A, infinity,
∴ [infinity is not in A="infinity is unbounded"] ∀A, infinity.

The word "infinity" is in A, but infinity is not in A, so (infinity in A) and (infinity not in A) make infinity empty in A. So therefore, the word "infinity" is a placeholder for ∅ in A, and ∅ in A has its consequence that negates A from being true.

Prove that the axiom "infinity is unbounded" is false.
Proof:
Note that infinity is not in the axiom A="infinity is unbounded". ∴infinity is empty in A.
Assume the axiom A="infinity is unbounded" is true. Therefore [infinity is unbounded]⇒[∅ is unbounded]⇒[∅ is not bounded]. But, ∅ belongs to ∅, so ∅ is bounded; [∅ is bounded] and [∅ is not bounded], ➔🡐, ∴A is false. Q.E.D.

We have proved that the "infinity is unbounded" is a false axiom. Inside an axiom, every element should hold for the whole axiom to hold, but infinity does not hold in the "infinity is unbounded" axiom. Worse yet, because "infinity is unbounded" is an axiom established by consensus, guarantees, and supports, the infinity from the axiom stands <u>for all</u> infinity! "Infinity is unbounded" is false, but the consensus and guarantees and supports support the otherwise. The "infinity is unbounded" is false, and because it is an established axiom, you must accept that the false is true. False is true. Confused? The false axiom leads to the existence of infinity being indeterminate when “infinity is infinity” strictly based on the false axiom without any further analysis.

Further analysis finds that the false ***infinity*** from the axiom ***is not*** the actual truly unbounded ***infinity***; false is not true. We can even further verify “infinity is not infinity ∀infinity” by the Definition of Existence, and thus “infinity is not infinity ∀infinity” stands true for all infinity from the “infinity is unbounded” axiom. Thank Huo Jian Hua for His Definition of Boundedness.

[Infinity is infinity] is a direct result from the false axiom, and the false axiom leads to the indeterminate existence of infinity because of its confusing “false is true”. Of course, “false is true” is not correct. [Infinity is not infinity ∀infinity] negates the truthfulness of the false infinity from the axiom. False is not true, no logical confusion, and this is correct. Thank Huo Jian Hua for His Definition of Boundedness.

Unless for a discussion purpose on infinity, infinity is not listed in a truth table because infinity is not an element, nor does infinity belong to a table. (The same applies for the unbounded.) However, for the proof discussion, infinity is in the Definition of Existence table. This infinity is a forced made element in the “infinity is unbounded” axiom, and by no surprise, a such infinity is not infinity, and this forced made infinity causes the “infinity is unbounded” axiom to be false.

Should we still be using the false axiom in mathematics? The “infinity is unbounded” axiom works for calculus, infinite series, infinite sequences, infinity norms, infinitary unions, and infinitary intersections, so why stop using it if it works to solve math problems? Therefore, we need to justify the choice of using infinity in mathematics, but how? We apply what the formal system gives us. The “infinity is unbounded” axiom in the formal system must stand no matter what. The “infinity is not infinity” supports the axiom both on-the-paper and off-the-paper while the axiom holds. The “infinity is infinity” not only creates a paradox, but it also leads the axiom to be torn down off-the-paper which means that the axiom does not hold. Therefore, we throw out the “infinity is infinity” to avoid the paradox, to hold on to the consensus and guarantees and supports for the “infinity is unbounded” axiom, along with the IINI∀I caveat, therefore, ∃∞∀∞, and thus to justify the use of infinity in mathematics based on this formal system. However, as always, all rights-benefits go to Huo Jian Hua by the guaranteed contract term of use on Huo Jian Hua’s Definition of Boundedness.

Nevertheless, the false axiom appears being the remedy for this twisted false-is-true world, where the true world is supposed to honors all rights-benefits to Huo Jian Hua by the contract guaranteed to Huo Jian Hua because Grandmaster’s guaranteed us — to prove – infinity – is not – infinity – serious! Now, by the guaranteed contract term of use on Huo Jian Hua’s Definition of Boundedness, it is your turn to honor Huo Jian Hua’s guaranteed contractual rights-benefits all the way up to infinity ∃∞∀∞ unbounded end. Honor Huo Jian Hua all His guaranteed rights-benefits, now! ~~~

OFF THE PAPER – ∃∞∀∞ by An Alternative System

To prove the existence of infinity using the Axiom of Existence: [∃x]⇔[x is not empty]∀x, instead of the one axiom system with the Definition of Existence, we'll have a two-axiom system while using Huo Jian Hua's Definition of Boundedness.

- Axiom of Infinity: Infinity is unbounded.

  ∴ Theorem A: Infinity is not bounded ∀infinity.

- Axiom of Existence: [∃x]⇔[x is not empty] ∀x.
- Huo Jian Hua's Definition of Boundedness: x is bounded if and only if x belongs to y ∀x, y.

Prove that infinity is not empty ∀infinity.

Proof: Suppose infinity is empty ∴infinity belongs to empty ∴infinity is bounded (by Huo Jian Hua's Definition of Boundedness). But, infinity is not bounded (by Theorem A), (infinity is not bounded) and (infinity is bounded) ⇒⇐ ∀infinity ∴infinity is not empty ∀infinity. ■

Verify infinity is not empty ∀infinity.
Proof: From the previous result, infinity is not bounded ∀infinity.

Suppose infinity is empty, so therefore empty is not bounded ∀empty. But, empty belongs to empty, so empty is bounded by Huo Jian Hua's Definition of Boundedness, (empty is bounded) and (empty is not bounded) ➔🡐 ∀infinity. ∴infinity is not empty ∀infinity. ■

Prove ∃infinity ∀infinity.

Proof: Infinity is not empty ∴∃infinity ∀infinity, by Axiom of Existence. ■

The proof for the existence of the natural numbers under this system, each natural number can be not empty, but each natural number needs to be proven not infinity because the induction process requires not infinity. The counter argument can argue that 1<2 where 2 is infinity (if you stand at infinity, 1 or 2 is infinity from you), and so on, so the induction process fails right there. To prove that each natural number is not infinity, the formal paper has used Huo Jian Hua's Definition of Bounded to do this because the natural numbers are abstractions rather than facts for being an axiom.

Hereinabove, we have applied two axioms "Infinity is unbounded" axiom and Axiom of Existence, and, Huo Jian Hua's Definition of Boundedness to prove ∃infinity ∀infinity. Of course, as always, for the use of Huo Jian Hua's Definition of Boundedness, all rights-benefits go to Huo Jian Hua, and, all rest of rights-benefits go to Huo Jian Hua by the guaranteed term of use on Huo Jian Hua's Definition of Boundedness because Grandmaster's guaranteed us.

文外 — 解開無限的一個悖論

此篇討論無限不確定的存在，並解開與它相關的一個悖論。當無限是無限時，無限呈現出一個悖論，由於〈一切無限皆非無限〉定理的緣故。〈無限即非無限〉與〈無限即無限〉之間哪一個是正確的？雖然數學並不經常證明已成立的定理，然而在面對如此悖論時我們卻必須在「無限即無界」公理系統於此文外篇證明「無限即無界」公理是虛假的，以解開該悖論。

證明無限即無限∀無限。

證： 根據對「無限即無界」的共識、保證及支持，重述[覽 A1]於以下

無限	無界	無限即無界	
是	是	是	[覽 A1]

當「無限即無界」公理支承時：

無限	無限	無限即無限	
是	是	是	[覽 A1.2]

∴無限即無限∀無限。證明完畢。

無限的不確定存在當無限即無限時。

我們已證明「無限即無限」當「無限即無界」公理支承時。重述[覽 E_N]於以下：

排 / 行	1	2	3	4	5	6	7	
1	x	∃x	x 即非∞	[∃x]⇔[x 即非∞]	∄x	x 即∅	[∄x]⇔[x 即∅]	
2	是	是	是	是	否	否	是	
3	否	是	是	是	否	否	是	
4	∅	是	是	是	是	是	是	
5	非∞	是	是	是	否	否	是	
6	∞	是	是	是	否	否	是	[覽 E_N]

〈「存在」的定義〉列在第 4 排以及〈「不存在」的定義〉被列在第 7 排。「無限即無限」這樣的狀況並沒有列在以上〈「存在」的定義〉內，也沒有列在以上〈「不存在」的定義〉內，由於無限並非∅。所以，當無限即無限時，以上定義不能確定這樣的無限存在或不存在，所以如此無限的存在是不確定的。難道這不就是無限存否問題會被長期爭論不休的原因嘛！

所以現在我們是否可以直接跳到「不確定就是無限」的結論？不然，不確定也不是無限，以及由於不確定不是無限，因此不確定可以存在這篇文章內討論。

證明不確定即非無限∀不確定。

證： 假設不確定即無限 ∴ 無限即一個容器。但，無限即非一個容器，（無限即非一個容器）又（無限即一個容器）➔🡐∀不確定∴不確定即非無限∀不確定。證明完畢。

證明∃不確定∀不確定。

證： 不確定即非無限∀不確定∴ ∃不確定。重述〈「存在」的定義〉：
∃x 若且唯若 x 即非∞ ∀x，及同等地，x 及非∞若且唯若∃x ∀x。
∴ ∃不確定若且唯若不確定即非∞∀不確定，
∴ 不確定即非∞若且唯若∃不確定∀不確定。證明完畢。

解開無限的一個悖論

我們已達到有兩個無限的對立結論：[無限即無限∀無限]與[無限即非無限∀無限]。哪一個才是正確的？雖然數學鮮少證明已成立的公理，但是我們在此面對手上有這樣的悖論，我們必須證明「無限即無界」公理是虛假的以解開這個悖論。

證明「無限即無界」公理是虛假的

證： 在此證明的是先證明於 A=「無限即無界」內的無限即空無。「無限」這個辭在 A 內是∅的佔位符號。因為∅在 A 內，所以∅即無界是一個矛盾而造成 A 被否定為是。

無限這個辭是在公理 A 內。無限在 A 內所以無限屬於 A 因為[x 在 y 內]⇔[x 屬於 y]∀x、y。

證明[x 在 y 內]⇔[x 屬於 y]∀x、y。
證：重述有界的定義[D1 覽表]。附加 [x 在 y 內]以及 [x 屬於 y]⇔[x 在 y 內]等排於以下：

x	y	x 屬於 y	x 即有界	[x 即有界]⇔[x 屬於 y]		x 在 y 內	[x 屬於 y]⇔[x 在 y 內]	[x 在 y 內]⇔[x 屬於 y]
是	是	是	是	是		是	是	是
是	否	否	否	是		否	是	是
是	∅	否	否	是		否	是	是
否	是	否	否	是		否	是	是
否	否	是	是	是		是	是	是
否	∅	否	否	是		否	是	是
∅	是	是	是	是		是	是	是
∅	否	是	是	是		是	是	是
∅	∅	是	是	是		是	是	是

[D2 覽表]

由於無限屬於 A 所以無限即有界，其後果就是無限不在 A 內如下證明：

證明無限即不在公理 A 之內，A=「無限即無界」，∀A、無限。
證： 狀況一：無限即不在 A 之內，此狀況證明完畢。
狀況二：假設無限即在 A 之內，A=「無限即無界」。
[無限即在A之內，A=「無限即無界」]⇒[無限在A之內，A=「無限即無界」]⇒[無限屬於A=「無限即無界」]
∴ [(無限)屬於(A=「無限即無界」)]⇒[無限即有界]。但，無限即非有界，(無限即非有界)又(無限即有界)是矛盾的 ∀A、無限，
∴ [無限即不在 A 之內，A=「無限即無界」] ∀A、無限。

無限這個詞是在 A 內，但無限又不在 A 內所以(無限在 A 內)又(無限不在 A 內)造成在 A 內的無限是空無的，所以無限在 A 內是∅的佔位符號。其後果就是 A 會被否定為真實的。

證明公理「無限即無界」是虛假的。
證：
註：無限即不在公理 A 之內，A=「無限即無界」，•無限在 A 之內即空洞的。
假設公理 A=「無限即無界」是真實的。因此，[無限即無界]•[∅即無界]•[∅即非有界]。
但，∅屬於∅，所以∅即有界根據定義，[∅即有界]又[∅即非有界]，➔🡐，•A 是虛假的。證明完畢。

我們已證明「無限即無界」是個虛假即非的公理。在一個公理裡面的每個元素都應該支承著才可使得整個公理得以支承著，但是無限在「無限即無界」內並沒有支承著。然而更糟的是，由於「無限即無界」是個既已經共識、保證及支持而成立的公理，從這個公理而來的無限還代表了全量無限哪！「無限即無界」是虛假即非的，然而於它的共識、保證及支持卻不以為然。縱使「無限即無界」是虛假即非的，然而卻因為它是個既已成立的公理，你必須接受其非即是。非即是（邏輯否即邏輯是），猶疑不？所以每當「無限即無限」僅僅建立在此虛假公理上且缺乏進一步分析時，此虛假公理導致其無限的存在是不確定的。不過，仍然要一如既往地根據霍建華〈有界的定義〉使用契約保證，一切權力益全都歸霍建華。

而進一步分析出在此公理內被否定的無限並非是真際上真正無界的無限，邏輯否即非邏輯是。我們並且可進一步根據〈存在的定義〉查對「無限即非無限∀無限」，一切如此從「無限即無界」公理來的「無限即非無限」得成立為正確的結論。感恩霍建華的〈有界的定義〉。

[無限即無限]是直接從虛假公理出來的結果，然而該虛假公理導致無限不確定的存在，由於那混淆的「邏輯否即邏輯是」，「邏輯否即邏輯是」當然不正確。然而[無限即非無限∀無限]否定從該公理出來的虛假無限的真實性，邏輯否即非邏輯是，沒有邏輯混淆，這樣才是正確的。感謝霍建華的〈有界的定義〉。

除非為了要達到對無限做討論的目的，無限是不會被列入任何一個真值表內。（無界亦是。）因為無限不是一個元素，所以無限也不該屬於任何一個覽表。然而為了要討論無限的證明，無限出現在〈存在的定義〉覽表內，於此的無限早已在「無限即無界」公理內就被牽強成了一個元素。但不意外的是這樣的無限即非無限，並且這樣被牽強的無限反而造成「無限即無界」公理成為虛假即非的。

我們是否應該繼續在數學裡使用該虛假公理？因為「無限即無界」公理可以應用在微積分學、無限級數、無限序列、無限範數、無限聯集以及無限交集，所以有必要停止使用它來解答數學問題嗎？因此，我們需要為數學中使用無限的選擇辯護，但該如何做到呢？我們應用正式系統所給予的內容。正式系統中的「無限即是無界」公理必須無論如何都成立。「無限不是無限」在此公理成立的情況下，不論在理論上或實務上都支持這一公理。然而，「無限是無限」不僅製造了一個悖論，在進一步的實務探討中還導致公理被「無限是無限」撕毀，意味著該公理不成立。因此，我們排除「無限是無限」以避免悖論，維持「無限即無界」公理的共識與保證及支持，並且結合 IINI∀I 的警告，從而得出∃∞∀∞，以此來證明在這正式系統中使用無限的正當性，而且還是一如既往地一切權力益全都歸霍建華根據霍建華〈有界的定義〉使用契約保證。

無論如何，看這樣子此虛假公理就是用來對治這個被錯置顛倒的世界，而原本應該的正常世界是根據霍建華的契約保證兌現一切權力益全都歸霍建華，因為濟公保證—要—證明—無限—不是—無限—是認真的！根據霍建華〈有界的定義〉使用契約保證，現在是換你要對霍建華認帳霍建華的契約保證霍建華的權力益無限∃∞∀∞無界到底。要認帳！現在就認帳！~~~

文外 – ∃∞∀∞ 由另一個系統

為了證明「無限」的存在經以「『存在』的公理」：[∃x]⇔[x 即非空無]∀x，在使用霍建華〈有界的定義〉的同時，我們將以雙公理系統取代單公理系統與其「『存在』的定義」。

- 「無限」的公理：無限即無界。∴ 定理 A：無限即非有界∀無限。
- 「『存在』的公理」：[∃x]⇔[x 即非空無]∀x。
- 霍建華〈有界的定義〉：x 即有界若且唯若 x 屬於 y ∀x、y。

證明無限即非空無∀無限。

證： 假設無限即空無 ∴無限屬於空無 ∴無限即有界（根據霍建華〈有界的定義〉）。但，無限即非有界（根據定理 A），（無限即非有界）又（無限即有界）⇒⇐ ∀無限 ∴無限即非空無 ∀無限。■

查對無限即非空無∀無限。

證： 從之前的結論：無限即非有界∀無限。

假設無限即空無，所以空無即非有界∀空無。但，空無屬於空無 ∴空無即有界，根據霍建華〈有界的定義〉，（空無即有界）又（空無即非有界）➔🡨 ∀無限 ∴無限即非空無∀無限。■

證明∃無限 ∀無限。

證： 無限即非空無 ∴∃無限 ∀無限，根據「『存在』的公理」。■

在這個系統下證明自然數的存在，每個自然數可以是非空的，但每個自然數需要被證明不是無限大的，因為歸納過程要求不是無限大的。反駁論點可辯稱 1<2，而 2 是無限…等等（如果你站在無限之處，那麼 1 或 2 相對於你而言也是無限遠），因此歸納過程在那裡就失敗了。為了證明每個自然數不是無限的，正式論文使用了霍建華〈有界的定義〉來進行這項工作，因為自然數是抽象概念，而不是可作為公理的事實。

於此上述，我們應用了兩個公理「無限即無界」公理及「『存在』的公理」，以及霍建華〈有界的定義〉以證明∃無限∀無限。當然，一如既往，由於使用霍建華〈有界的定義〉的緣故，一切權力益全都歸霍建華，以及，一切剩下的權力益全都歸霍建華，根據霍建華〈有界的定義〉使用契約保證，因為濟公保證。大家現在就認帳!~~~~~~