NONEXISTENCE, EXISTENCE, ∃∞∀∞, AND ∃∅⟺∄∅

This section first discusses example situations when infinity (denoted by ∞) does not exist. However, to prove ∃∞∀∞ (there exists infinity for all infinity), we need the Definition of Existence: ∃x ⟺ x is not ∞∀x. (Seriously, thank Huo Jian Hua for Huo Jian Hua's sake. "Not ∞" is the essence for existence because now it has been His idea!) However, to prove the Definition of Existence, we need the Definition of Container, Definition of Element, and Huo Jian Hua's Definition of Boundedness. The approach allows us to later verify the IINI∀I. The Definition of Existence together with the Definition of Nonexistence can prove ∃∅⟺∄∅. In this section, we will also apply [x is y]⇒[x belongs to y]∀x, y proved by [Table XY1] from the previous section. Because the use of the Huo Jian Hua's Definition of Boundedness in the IINI∀I proof, all rights-benefits belong to 霍建華 (Huo Jian Hua) by His contract guarantee. 4

Why prove ∃∞∀∞?

It is because mathematics uses infinity. If ∞ does not exist, then ∞ is ∅, by the following Definition of Nonexistence:

∄x if and only if x is ∅ ∀x.

Prove the Definition of Nonexistence: [∄x]⟺[x is ∅] ∀x.

Proof: Let ∅ denote empty.

x	∄x	x is ∅	[∄x]⟺[x is ∅]
T	F	F	T
F	F	F	T
∅	T	T	T

[Table NE]

x	∄x	x is ∅	[∄x]⟺[x is ∅]
∅	T	T	T

Q.E.D.

Therefore, if infinity does not exist, improper integrals, infinite series, and infinite sequences can lead to empty or undefined. For example, the solution of the following improper integral is 1∈ℝ:

$$\int_1^{\infty} x^{-2}dx = -x^{-1}|_1^{\infty} = \frac{-1}{x}|_1^{\infty} = [\lim_{x\to\infty}\frac{-1}{x}] - \frac{-1}{1} = 0 + 1 = 1.$$

Suppose that infinity does not exist, then the above integral becomes:

$$\int_1^{\varnothing} x^{-2}dx = -x^{-1}|_1^{\varnothing} = \frac{-1}{x}|_1^{\varnothing} = \frac{-1}{\varnothing} - \frac{-1}{1} = \frac{-1}{\varnothing} + 1\,.$$

The simplification of $\frac{-1}{\varnothing} + 1$ into a real number is empty in ℝ. Therefore $\frac{-1}{\varnothing} + 1$ is empty in ℝ, as opposed to the nonempty solution $\int_1^{\infty} x^{-2}dx = 1$ in ℝ.

Consider the series $S = \sum_{n=0}^{\infty}\frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$. Therefore, $\frac{S}{2} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$. Add 1 on both side of the equal sign $1 + \frac{S}{2} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$, thus $1 + \frac{S}{2} = S$, and therefore $S = 2$. Suppose infinity does not exist, then $S = \sum_{n=0}^{\varnothing}\frac{1}{2^n}$. The $\sum_{n=0}^{\varnothing}\frac{1}{2^n}$ expansion involves $(\frac{1}{2^0} + \frac{1}{2^{\varnothing}}) = (1 + \frac{1}{2^{\varnothing}})$, so S = $1 + \frac{1}{2^{\varnothing}} + \cdots$. Therefore, S is empty in ℝ, as opposed to the nonempty $S = \sum_{n=0}^{\infty}\frac{1}{2^n} = 2$ in ℝ.

Consider the sequence $a_{k+1} = a_k + a_{k-1}$, where $a_0 = 0, a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 3, a_5 = 5, a_6 = 8, \ldots$. Therefore, $(\frac{a_{k+1}}{a_k})_{k=1}^{\infty} = \left(1,2,\frac{3}{2},\frac{5}{3},\frac{8}{5},\ldots\right)$, $\lim_{k\to\infty}\frac{a_{k+1}}{a_k} = \frac{1+\sqrt{5}}{2}$. If infinity does not exist, then $(\frac{a_{k+1}}{a_k})_{k=1}^{\varnothing}$, $\lim_{k\to\varnothing}\frac{a_{k+1}}{a_k} = \frac{a_\varnothing}{a_\varnothing} \stackrel{?}{=} 1$ being undefined, as opposed to $\lim_{k\to\infty}\frac{a_{k+1}}{a_k} = \frac{1+\sqrt{5}}{2}$.

Recapture and contrast the above examples:

$$\left(\int_1^{\infty} \mathrm{x}^{-2}\mathrm{dx} = 1\right) \textit{verses} \left(\int_1^{\varnothing} \mathrm{x}^{-2}\mathrm{dx} = \varnothing\right)$$

$$\left(S = \sum_{n=0}^{\infty}\frac{1}{2^n} = 2\right) \textit{verses} \left(\sum_{n=0}^{\varnothing}\frac{1}{2^n} = \varnothing\right)$$

$$\left(\lim_{k\to\infty}\frac{a_{k+1}}{a_k} = \frac{1+\sqrt{5}}{2}\right) \textit{verses} \left(\lim_{k\to\varnothing}\frac{a_{k+1}}{a_k} = \frac{a_\varnothing}{a_\varnothing} \stackrel{?}{=} 1\right)$$

The $\int_1^{\infty} \mathrm{x}^{-2}\mathrm{dx} = 1$, $\sum_{n=0}^{\infty}\frac{1}{2^n} = 2$, *and*, $\lim_{k\to\infty}\frac{a_{k+1}}{a_k} = \frac{1+\sqrt{5}}{2}$ show that mathematics uses infinity for nonempty solutions, as opposed to infinity being empty, meaning that infinity does not exist, causing undefined operations for $\mathbb{R}$, or empty in $\mathbb{R}$. Therefore, mathematics needs a formal proof for the existence of infinity. [∞ is not ∞ ∀∞] is equivalent to justify [∃∞∀∞] by the Definition of Existence, and, [∞ is not ∅ ∀∞] from previous proof shows that the existence of ∞ is nontrivial. This section further provides the derivation for the Definition of Existence by mathematical proofs.

Definition of Container

C is a container if and only if x in C ∀x, C.

Prove the Definition of Container: [C is a container]⇔[x in C] ∀x, C.

Proof: By the given being a definition, "C is a container" is "C is a container of x" ∴

C	x	C is a container	x in C	[C is a container]⇔[x in C]
T	T	T	T	T
T	F	F	F	T
T	∅	T	T	T
F	T	F	F	T
F	F	T	T	T
F	∅	T	T	T
∅	T	F	F	T
∅	F	F	F	T
∅	∅	T	T	T

[Table C]

For x=T, and, y=T

C	x	C is a container	x in C	[C is a container]⇔[x in C]
T	T	T	T	T

[Table C_1]

Q.E.D.

Containers

Truth T, Falsehood F, empty ∅, and x are containers. However, infinity is not a container, for the "infinity is unbounded" axiom to hold.

Prove x in x ∀x.

Proof: By [Table EQ](3) from Supporting Proofs for Empty, [x=x]⇒[x in x] ∀x. Q.E.D

Prove that Truth T is a container ∀T.

Proof: [T=T]⇒[T in T] ∴T is a container of T, ∴T is a container ∀T. Q.E.D.

Prove that Falsehood F is a container ∀F.

Proof: [F=F]⇒[F in F] ∴F is a container of F, ∴F is a container ∀F. Q.E.D.

Prove that ∅ is a container ∀∅.

Proof: ∅ in ∅, ∴ ∅ is a container of ∅, ∴ ∅ is a container ∀∅. Q.E.D.

Prove that x is a container ∀x.

Proof: [x=x]⇒[x in x] ∴x is a container ∀x.

x	x in x	[x is a container]⇔[x in x]	x is a container
T	T	T	T
F	T	T	T
∅	T	T	T

[Table C_x]

Each entry under [x in x] column is [T] ∴ each entry under [[x is a container]⇔[x in x]] column is [T] by definition, ∴ each entry under [x is a container] column solves to [T] ∴ x is a container ∀x. Q.E.D.

Prove [x is y] ⇒ [y is a container] ∀x, y.

Proof:

x	y	x is y	x in y	y is a container (of x)	[x is y] ⇒ [y is a container]
T	T	T	T	T	T
T	F	F	F	F	T
T	∅	F	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	∅	F	F	F	T
∅	T	F	T	T	T
∅	F	F	T	T	T
∅	∅	T	T	T	T

[Table C_2]

Q.E.D.

Prove that **infinity** is not a container ∀infinity.

Proof: Note that [x is y] ⇒ [x belongs to y] ∀x, y.

Suppose infinity is a container ∴ infinity belongs to a container ∴ infinity is bounded, by Huo Jian Hua's Definition of Boundedness. But, infinity is not bounded; (infinity is not bounded) and (infinity is bounded) ➔🡐∀ infinity ∴ infinity is not a container ∀infinity. Q.E.D.

The Essence for Existence -- Not ∞.

With using Huo Jian Hua's Definition of Boundedness, we have proved that ∞ is not a container ∀∞. With using the Definition of Container, the followings prove that Truth T, falsehood F, ∅, x, or (not ∞) are not ∞. The following (not ∞)s are prepared for the Definition of Existence truth table entries:

Prove that Truth T is not ∞ ∀T.

Proof: Suppose T is ∞ ∴∞ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 ∀T, ∴ T is not ∞ ∀T. Q.E.D.

Prove that Falsehood F is not ∞ ∀F.

Proof: Suppose F is ∞ ∴∞ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 ∀F, ∴ F is not ∞ ∀F. Q.E.D.

Prove that ∅ is not ∞ ∀∅.

Proof: Suppose ∅ is ∞ ∴∞ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 ∀∅, ∴ ∅ is not ∞ ∀∅. Q.E.D.

Prove that not ∞ is not ∞ ∀not ∞.

Proof: Suppose not ∞ is ∞ ∴∞ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 ∀not ∞ , ∴not ∞ is not ∞ ∀not ∞. Q.E.D.

Definition of Element

(Element x≡x)∈C if and only if x in C ∀x, C.

Prove the Definition of Element: [(Element x≡x) ∈C]⇔[x in C] ∀x, C.

Proof:

C	x	x∈C	x in C	[x∈C]⇔[x in C]
T	T	T	T	T
T	F	F	F	T
T	∅	T	T	T
F	T	F	F	T
F	F	T	T	T
F	∅	T	T	T
∅	T	F	F	T
∅	F	F	F	T
∅	∅	T	T	T

[Table EL]

For x=T, and, y=T

C	x	x∈C	x in C	[x∈C]⇔[x in C]
T	T	T	T	T

[Table EL_1]

Q.E.D.

Elements

Truth T, Falsehood F, ∅, and x are elements. However, infinity is not an element, for the “infinity is unbounded” axiom to hold.

Prove that Truth T is an element ∀T.

Proof: T is a container of T∀T, ∴T in T ∴element T≡T∈T, ∴ T is an element ∀T. Q.E.D.

Prove that Falsehood F is an element ∀F.

Proof: F is a container of F ∀F, ∴F in F ∴element F≡F∈F, ∴ F is an element ∀F. Q.E.D.

Prove ∅ is an element ∀∅.

Proof: ∅ in ∅ already ∴element ∅≡∅∈∅ ∴∅ is an element ∀∅. Q.E.D.

Prove x∊x ∀x.

Proof:

x	x in x	[x∈x]⇔[x in x]	x∊x
T	T	T	T
F	T	T	T
∅	T	T	T

[Table EL_x]

Each entry under [x in x] column is [T] ∴ each entry under [[x∈x]⇔[x in x]] column is [T] by definition, ∴ each entry under [x∊x] column solves to [T] ∴ x∊x ∀x. Q.E.D.

Prove that infinity is not an element ∀infinity.

Proof: Suppose infinity is an element ∴ infinity belongs to an element ∴ infinity is bounded, by Huo Jian Hua's Definition of Boundedness. But, infinity is not bounded; (infinity is not bounded) and (infinity is bounded) ➔🡐∀infinity ∴infinity is not an element ∀infinity. Q.E.D.

Prove [x∈C] ⇔ [x belongs to C] ∀x,C.

Proof:

All entries in column [x in C]⇔[x∈C] is [T] by definition:

x	C	[x in C]⇔[x∈C]	x in C	x∈C	x belongs to C	[x∈C]⇔[x belongs to C]
T	T	T	T	T	T	T
T	F	T	F	F	F	T
T	∅	T	F	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T
F	∅	T	F	F	F	T
∅	T	T	T	T	T	T
∅	F	T	T	T	T	T
∅	∅	T	T	T	T	T

[Table C_3]

Q.E.D.

Definition of Existence

∃x if and only if x is not ∞∀x.

(Note: "∃" denotes the existential quantifier. "∀" denotes the universal quantifier.)

Prove the Definition of Existence: [∃x]⇔[x is not ∞] ∀x.

Proof:

row \ col	1	2	3	4
1	x	∃x	x is not ∞	[∃x]⇔[x is not ∞]
2	T	T	T	T
3	F	T	T	T
4	∅	T	T	T
5	not ∞	T	T	T
6	∞	T	T	T

[Table E1]

The earlier proofs have shown that there exist T, F, and ∅ being elements, and thus [Table E1] lists the elements in column 1.

Therefore, (row 2, col 2), (row 3, col 2), and (row 4, col 2) are [T] entries, respectively. T, F, and ∅ in column 1 of [Table E1] are not ∞, so the [T] entries are in (row 2, col 3), (row 3, col 3), and (row 4, col 3) respectively. The (row 5, col 2) is [T], for there exists T, F, or ∅ which is not ∞ from the [Table E1] itself. "Not ∞ is not ∞" results [T] in (row 5, col 3). Infinity is in the "infinity is unbounded" axiom, so therefore, by definition, infinity is an element of the "infinity is unbounded" axiom, and thus this infinity is an element. Because the infinity from the "infinity is unbounded" axiom is an element, an element that can also be in (row 6, col 1), denoted by ∞, such infinity is not infinity which is to be shown in the following paragraph, so we can further examine if the (row 6, col 1) entry works for the Definition of Existence proof.

[∃infinity] ⇒ [infinity is not infinity]:

By consensus and guarantees and supports to the "infinity is unbounded" axiom, while the "infinity is unbounded" axiom holds <u>∀infinity</u>, there exists infinity from within the "infinity is unbounded" axiom, so therefore (row 6, col 2) is [T]. However, a such infinity in the "infinity is unbounded" is not infinity. The infinity in "infinity is unbounded" is an element ∴infinity is an element. But, infinity is not an element; (infinity is not an element) and (infinity is an element), so the infinity being an element in the "infinity is unbounded" is empty, and empty is not infinity ∴ the infinity being a placeholder for empty in the "infinity is unbounded" is not infinity ∴ [∃infinity] ⇒ [infinity is not infinity].

[infinity is not infinity] ⇒ [∃infinity]:

Whether [infinity is not infinity] is true or false, by consensus and guarantees and supports to the "infinity is unbounded" axiom, while the "infinity is unbounded" axiom holds, there exists infinity from within the "infinity is unbounded" axiom <u>∀infinity</u> ∴ [infinity is not infinity] ⇒ [∃infinity].

∴[∃infinity]⇔[infinity is not infinity] ∴ the entry in (row 6, col 4) of [Table E1] is [T]. Because (row 6, col 2) is [T], (row 6, col 3) must be [T] for (row 6, col 4) to hold [T].

Every row entry in column 2 is [T], and every row entry in column 3 is [T], ∴ every row entry in column 4 is [T] from [Table E1] ∀x ∴ [∃x]⇔[x is not ∞] ∀x. Q.E.D.

*Prove **∃∞∀∞**. (Prove that there exists infinity for all infinity.)*

Proof:

The previous section has proved that ∞ is not ∞ ∀∞. ∴ ∞ is not ∞ ∴∃∞.

Restate the Definition of Existence: [∃ x]⇔[x is not ∞] ∀x ∴[x is not ∞]⇔[∃ x] ∀x.

∴[∃∞]⇔[∞ is not ∞] ∀∞ ∴[∞ is not ∞]⇔[∃∞] ∀∞ ∴∃∞∀∞. Q.E.D.

*Verify **∃∞∀∞**.*

Proof:

The (row 6, col 3) entry in [Table E1] solves to [T] ∴ ∞ is not ∞ ∴∃∞.

Restate the Definition of Existence: [∃ x]⇔[x is not ∞] ∀x ∴[x is not ∞]⇔[∃ x] ∀x.

∴[∃∞]⇔[∞ is not ∞] ∀∞ ∴[∞ is not ∞]⇔[∃∞] ∀∞ ∴∃∞∀∞. Q.E.D.

From ∞ is not ∅ ∀∞ in the previous section, and ∃∞∀∞, the existence of ∞ is nontrivial, so per contractual guaranteed terms of use for Huo Jian Hua's Definition of Boundedness, all rights-benefits of ∞ (infinity) belong to Huo Jian Hua, and, because Grandmaster's guaranteed us to prove that infinity is not infinity, seriously.

Verify ∞ is not ∅ ∀∞.

Proof: Suppose ∞ is ∅, the "infinity is unbounded" axiom becomes "∅ is unbounded" ∴unbounded is a container, ∴unbounded belongs to a container ∴unbounded is bounded, by Huo Jian Hua's Definition of Boundedness. But, unbounded is not bounded; (unbounded is not bounded) and (unbounded is bounded) ➔⟵ ∀∞ ∴∞ is not ∅ ∀∞. Q.E.D.

Verify ∞ is not ∞ ∀∞.

Proof: ∵∃∞∀∞ ∴∃∞ ∴ [∃∞]⇔[∞ is not ∞] ∀∞ ∴ ∞ is not ∞ ∀∞. Q.E.D.

Prove ∃unbounded ∀unbounded. (Prove that there exists unbounded for all unbounded.)

Proof:

Suppose unbounded is ∞ ∴ ∞ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔⟵ ∀unbounded ∴ unbounded is not ∞ ∀unbounded ∴∃unbounded ∀unbounded. Q.E.D.

Verify ∃unbounded ∀unbounded.

Proof: From [Table B1] in the "Prove that Infinity is not Infinity" section, by consensus and guarantees and supports to the "infinity is unbounded" axiom, while the "infinity is unbounded" axiom holds ∀unbounded, there exists unbounded from within the "infinity is unbounded" axiom. ∴∃unbounded ∀unbounded. Q.E.D.

Therefore, all rights-benefits of the unbounded belong to Huo Jian Hua ∀unbounded by His contract guaranteed to Huo Jian Hua.

Prove that unbounded is not ∅ ∀unbounded.

Proof:

Suppose unbounded is ∅ ∴unbounded belongs to ∅ ∴unbounded is bounded. But, unbounded is not bounded; (unbounded is not bounded) and (unbounded is bounded) ➔🡐 ∀unbounded ∴ unbounded is not ∅ ∀unbounded. Q.E.D.

Verify that unbounded is not ∅ ∀unbounded.

Proof:

Suppose unbounded is ∅ ∴"infinity is unbounded" axiom becomes "infinity is ∅", so ∞ is ∅. But, ∞ is not ∅; (∞ is not ∅) and (∞ is ∅) ➔🡐 ∀unbounded. ∴unbounded is not ∅ ∀unbounded. Q.E.D.

∃unbounded ∀unbounded, and unbounded is not ∅ ∀unbounded. Therefore, the existence of unbounded is not trivial. All unbounded rights-benefits belong to Huo Jian Hua by His contract guarantee. Note that unbounded is not infinity for all unbounded.

*Prove that unbounded is not **infinity** ∀unbounded.*

Proof: Suppose unbounded is infinity ∴ unbounded belongs to infinity ∴unbounded is bounded, by Huo Jian Hua's Definition of Boundedness. But, unbounded is not bounded; (unbounded is not bounded) and (unbounded is bounded) ➔🡐 ∀unbounded ∴unbounded is not infinity ∀unbounded. Q.E.D.

Verify that unbounded is not infinity ∀unbounded.

Proof: ∃unbounded ∀unbounded from the previous proof, ∴unbounded is not infinity by definition. Q.E.D.

Therefore, by the guaranteed contractual term of use for Huo Jian Hua's Definition of Boundedness, all unbounded rights-benefits belong to Huo Jian Hua, because The Grandmaster's guaranteed us to prove that infinity is not infinity, seriously.

Prove that x is not ∞ ∀x.

Proof: Suppose x is ∞ ∴∞ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 ∀x, ∴x is not ∞ ∀x. Q.E.D.

Verify that x is not ∞ ∀x.

Proof: From [Table E1], all truth table values are [T] under [x is not ∞] at column 3 for all values of x at column 1. ∴x is not ∞ ∀x. Q.E.D.

Prove ∃x ∀x.

Proof: x is not ∞ ∀x ∴x is not ∞ ∴∃x∀x. Q.E.D

Verify that ∃x∀x.

Proof:

From [Table E1], all truth table values are [T] under [∃x] at column 2 for all values of x at column 1. ∴∃x∀x. Q.E.D.

It is worth to note that the existence of x can be trivial because x can be ∅, yet the x does not exist. For this situation, the section of the paper will later prove ∃∅⟺∄∅ ∀∅.

Prove ∃…et cetera ∀…et cetera.

Proof: Suppose …et cetera is ∞ ∴ ∞ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 ∀…et cetera ∴ …et cetera is not ∞ ∀…et cetera ∴∃…et cetera ∀…et cetera. Q.E.D.

∃…et cetera *∀*…et cetera*.* All …et cetera rights-benefits belong to Huo Jian Hua by His contract guarantee. Note that …et cetera is not infinity for all …et cetera.

Infinity in the Definition of Nonexistence

By applying the Definition of Nonexistence, this section finds that “infinity doesn’t exist” is false. By combining [Table E1] from Definition of Existence proof, and [Table NE] from Definition of Nonexistence, the following [Table E_N] fills the shaded cells which are the cells missing from the [Table NE]:

row \ col	1	2	3	4	5	6	7
1	x	∃x	x is not ∞	[∃x]⬄[x is not ∞]	∄x	x is ∅	[∄x]⬄[x is ∅]
2	T	T	T	T	F	F	T
3	F	T	T	T	F	F	T
4	∅	T	T	T	T	T	T
5	not ∞	T	T	T	F	F	T
6	∞	T	T	T	F	F	T

[Table E_N]

The (row 5, col 5) states “there does not exist (not infinity) ∀(not infinity)”, so for the counterexample, [T] is not infinity, and [T] exists by Definition of Existence, so therefore (row 5, col 5) is false [F].

The (row 5, col 6) states “not infinity is empty ∀(not infinity)”, so for the counterexample, infinity is not infinity, and infinity not empty, so therefore (row 5, col 6) is false [F].

The (row 6, col 5) states “there does not exist infinity ∀infinity”, and the statement is false [F] because there is the ∃∞∀∞ proof.

The (row 6, col 6) states “infinity is empty ∀infinity”, and the statement is false [F] because there is the “infinity is not empty” proof, or the “infinity is unbounded” axiom would not hold.

Both (row 5, col 7) and (row 6, col 7) are true [T] from the results of column 5, and, column 6. All values in column 7 are [T], so therefore [∄x]⬄[x is ∅]∀x still holds true while [not ∞], and, [∞] rows are in the [Table E_N].

The [row 6] is entirely on infinity. So, when x≡∞, [∄x]⬄[x is ∅] becomes [∄∞]⬄[∞ is ∅]. However, according the [Table E_N], [∄∞] is false which negates the nonexistence of infinity, and it is because [∞ is ∅] is false, for ∞ is not ∅, otherwise the “infinity is unbounded” axiom would not hold. So therefore, ∄∞ being false negates the “infinity doesn’t exist” claim while the “infinity is unbounded” axiom (by consensus, with guarantees and supports) holds. Thank Huo Jian Hua for His Definition of Boundedness.

∃∅⇔∄∅

Both ∄∅∀∅, and, ∃∅∀∅ are valid statements. ∄∅∀∅ is by the Definition of Nonexistence, and ∃∅∀∅ is by the Definition of Existence. An example of ∄∅, let x=(π∈ℕ). But, π∈ℕ is ∅ ∴ x is ∅ ⇒ ∄x ∴∄(π∈ℕ)

∴∄(π∈ℕ)⇔(π∈ℕ) is ∅ ∀(π∈ℕ),

∴∄(∅)⇔(∅) is ∅ ∀(∅).

∴∄∅ ⇔ ∅ is ∅ ∀∅. ∅ is already empty, so ∄∅. To prove ∄∅∀∅, recall from the Definition of Nonexistence.

Prove ∄∅∀∅.

Proof: Definition of Nonexistence: [∄x]⇔[x is ∅]∀x.
∅ is ∅ ∴ ∄∅∀∅ by definition. Q.E.D.

However, ∅ does exist because ∅ is not ∞ although the existence of ∅ is trivial. To prove ∃∅∀∅, recall from the Definition of Existence.

Prove ∃∅∀∅.

Proof: Definition of Existence: [∃x]⇔[x is not ∞]∀x.

C denotes a container. Suppose ∅ is ∞ ∴ ∞ is C. But, ∞ is not C; (∞ is not C) and (∞ is C) ➔⬅ ∀∅ ∴ ∅ is not ∞ ∴ ∃∅∀∅. Q.E.D.

Prove ∃∅⇔∄∅ ∀∅.

Proof:

The [row 4] of [Table E_N] is entirely on ∅, so the row 4 is for all ∅.
The (row 4, col 2) is [T] and states “there exists ∅”, ∃∅;
The (row 4, col 5) is [T] and states “there does not exist ∅”, ∄∅.
∃∅ being T and ∄∅ being T are from the same row 4, ∴∃∅⇔∄∅ ∀∅. Q.E.D.

The existence of ∅ is trivial in which it becomes a nonexistent: ∃∅⇔∄∅∀∅. As for ∞, if ∞ were ∅, then the existence of ∞ would be trivial. Instead, we have proved that ∞ is not ∅ from the previous section, and we have further proved that ∃∞∀∞, so we can conclude that the existence of ∞ is nontrivial. As we were assuming that infinity was empty, thus a trivial existence, contradicting the nontrivial existence conclusion for infinity, this explains why the improper integral, infinite series, and infinite sequence would lead to the nonsensical empty or undefined. Thank Huo Jian Hua for His Definition of Boundedness, and all rights-benefits belong to Huo Jian Hua by His contract guarantee.

Prove ∃(π∈ℕ)⇔∄(π∈ℕ) ∀(π∈ℕ).

Proof:

(1) Suppose π∈ℕ is not ∅∴ π in ℕ. But, π not in ℕ; (π not in ℕ) and (π in ℕ) ➔⬅ ∀π∈ℕ ∴π∈ℕ is ∅ ∴∄(π∈ℕ).
(2) Suppose (π∈ℕ) is ∞ ∴∞ is a container ∴∞ belongs to a container ∴∞ is bounded, by Huo Jian Hua’s Definition of Boundedness. But, ∞ is not bounded; (∞ is not bounded) and (∞ is bounded) ➔⬅ ∀(π∈ℕ) ∴(π∈ℕ) is not ∞ ∴∃(π∈ℕ).

∃(π∈ℕ) ⇔ ∃∅; ∄∅⇔∄(π∈ℕ). ∴
∃(π∈ℕ) ⇔ ∃∅ ⇔ ∄∅ ⇔ ∄(π∈ℕ).
∴ ∃(π∈ℕ) ⇔ ∄(π∈ℕ) ∀(π∈ℕ). Q.E.D.

The existence of π∈ℕ is trivial in which it become a nonexistent as in [∃(π∈ℕ)]⇔[∄(π∈ℕ)].

In mathematics, you sometimes find your "solution" being infinity, and your math teacher tells you that the answer does not exist, or infinity is not a number (infinity is not even itself). Often enough the math problem is to tell you to find the answer from a number set, e.g., the real number set ℝ. Infinity is not in the real number set, so the answer is empty in ℝ, and therefore the answer does not exist in ℝ, i.e., ∄∞∈ℝ∀∞∈ℝ. Because ∞∈ℝ is ∅, the following proof uses ∃∅⇔∄∅∀∅.

Prove ∃(∞∈ℝ)⇔∄(∞∈ℝ) ∀(∞∈ℝ).

Proof:

Suppose ∞∈ℝ is not ∅. ∴∞ must be a nonempty element in ℝ ∴ (∞∈ℝ) ⇒ (∞ belongs to ℝ) ∴∞ is bounded, by Huo Jian Hua’s Definition of Boundedness. But, ∞ is not bounded; (∞ is not bounded) and (∞ is bounded) ➔⬅ ∀∞∈ℝ. ∴ ∞∈ℝ is ∅ ∴∃(∞∈ℝ)⇔∄(∞∈ℝ) ∀(∞∈ℝ). Q.E.D.

To verify ∄∞∈ℝ ∀∞∈ℝ, restate the Definition of Nonexistence:

[∄x]⇔[x is ∅]∀x, or , [x is ∅] ⇔ [∄x]∀x.

Let x denote (∞∈ℝ). But, (∞∈ℝ) is empty. ∴ x is ∅ ∴ ∄x ∀x ∴∄∞∈ℝ ∀∞∈ℝ.

The following proof is to verify $\exists\infty\in\mathbb{R}\ \forall\infty\in\mathbb{R}$.

Prove $\exists\infty\in\mathbb{R}\ \forall\infty\in\mathbb{R}$.

Proof: Suppose $\infty\in\mathbb{R}$ is ∞ $\therefore\infty$ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 $\forall\infty$ $\therefore$ $\infty\in\mathbb{R}$ is not ∞ $\therefore$ $\exists\infty\in\mathbb{R}\ \forall\infty\in\mathbb{R}$ by the Definition of Existence. Q.E.D.

The $\infty\in\mathbb{R}$ is a placeholder for $\varnothing$. The existence of $\infty\in\mathbb{R}$ is trivial in which it become a nonexistent as in $[\exists(\infty\in\mathbb{R})]\Leftrightarrow[\nexists(\infty\in\mathbb{R})]$.

Infinity is Not a Number.

Prove that infinity is not a number $\forall$infinity.

Proof: Suppose infinity is a number ∴infinity belongs to a number ∴infinity is bounded, by Huo Jian Hua's Definition of Boundedness. But, infinity is not bounded; (infinity is not bounded) and (infinity is bounded) ➔🡐 $\forall$infinity ∴ infinity is not a number $\forall$infinity. Q.E.D.

Prove that infinity is not y $\forall$infinity.

Proof: Suppose infinity is y ∴infinity belongs to y ∴infinity is bounded, by Huo Jian Hua's Definition of Boundedness. But, infinity is not bounded; (infinity is not bounded) and (infinity is bounded) ➔🡐 $\forall$infinity. ∴ infinity is not y $\forall$infinity. Q.E.D.

Prove that infinity is not y $\forall$y.

Proof: Suppose infinity is y ∴infinity belongs to y ∴infinity is bounded, by Huo Jian Hua's Definition of Boundedness. But, infinity is not bounded; (infinity is not bounded) and (infinity is bounded) ➔🡐 $\forall$y. ∴ infinity is not y $\forall$y. Q.E.D.

Prove that infinity is not y $\forall$infinity, y.

Proof: Suppose infinity is y ∴infinity belongs to y ∴infinity is bounded, by Huo Jian Hua's Definition of Boundedness. But, infinity is not bounded; (infinity is not bounded) and (infinity is bounded) ➔🡐 $\forall$infinity, y. ∴ infinity is not y $\forall$infinity, y. Q.E.D.

In general, infinity does not exist in any container C, i.e., $\nexists\infty\in C\ \forall\infty\in C$. The existence of $\infty\in C$ is trivial in which it become a nonexistent as stated in the following $[\exists(\infty\in C)]\Leftrightarrow[\nexists(\infty\in C)]$ proof.

Prove $\exists(\infty\in C)\Leftrightarrow\nexists(\infty\in C)\ \forall(\infty\in C)$.

Proof: Suppose $\infty\in C$ is not $\varnothing$ $\therefore\infty$ must be a nonempty element in C $\therefore(\infty\in C)\Rightarrow(\infty$ belongs to C) by [Table C_3] $\therefore\infty$ is bounded, by Huo Jian Hua's Definition of Boundedness. But, ∞ is not bounded; (∞ is not bounded) and (∞ is bounded) ➔🡐 $\forall\infty\in C$ $\therefore$ $\infty\in C$ is $\varnothing$ $\therefore\exists(\infty\in C)\Leftrightarrow\nexists(\infty\in C)\ \forall(\infty\in C)$. Q.E.D.

Verify $\exists\infty\in C\ \forall\infty\in C$.

Proof: Suppose $\infty\in C$ is ∞ $\therefore\infty$ is a container. But, ∞ is not a container; (∞ is not a container) and (∞ is a container) ➔🡐 $\forall\infty$ $\therefore$ $\infty\in C$ is not ∞ $\therefore$ $\exists\infty\in C\ \forall\infty\in C$ by the Definition of Existence. Q.E.D.

$\infty\in C$ is a placeholder for $\varnothing$ because $\nexists\infty\in C$.

Verify $\nexists\infty\in C\ \forall\infty\in C$.

Proof: Suppose $\exists\infty\in C$ $\therefore\infty$ belongs to C by [Table C_3] $\therefore\infty$ is bounded, by Huo Jian Hua's Definition of Boundedness. But, ∞ is not bounded; (∞ is not bounded) and (∞ is bounded) ➔🡐 $\forall\infty\in C$ $\therefore$ $\nexists\infty\in C\ \forall\infty\in C$. Q.E.D.

In this section, we have proved $\exists\infty\forall\infty$, verified IINI∀I theorem, and proved $\nexists\varnothing\Leftrightarrow\exists\varnothing$ with the use of Huo Jian Hua's Definition of Boundedness, Definition of Nonexistence, and Definition of Existence. Through using Huo Jian Hua's Definition of Boundedness, we have proved that infinity is not a number. The $\exists(\infty\in C)\Leftrightarrow\nexists(\infty\in C)\ \forall(\infty\in C)$ result says that infinity is a nonexistent everywhere in any container C (e.g., the observable universe being a container), and a such infinity is left-out rest from any research paper because the paper would be a container, yet the Definition of Nonexistence in [Table E_N] still negates "infinity does not exist" claim, and ∃∞∀∞ advocates the existence of infinity from the "infinity is unbounded" axiom by the consensus and guarantees and supports. Why? The infinity in ∃∞∀∞ that can exist on paper is based on the written infinity from the "infinity is unbounded" axiom, and the infinity in ∃∞∀∞ is not the infinity that cannot be in any research paper.

"Does Infinity exist in the universe?" It is like asking, "Does $\boldsymbol{\pi} \in \mathbb{R}$ exist in the natural number universe?" The answer is no. Whatever the $\boldsymbol{\pi}$ that you present to the natural number world would appear either empty or not the $\boldsymbol{\pi}$ itself, and thus $\boldsymbol{\pi}$ appears to be a nonexistent in the universe of natural numbers; analogously, the infinity presented in this universe is not the infinity itself, and it appears empty, and thus infinity appears to be a nonexistent in the universe. It might be easier by interpreting "∞ is not ∞ ∀∞" as: the infinity from the "infinity is unbounded" axiom is not the actual infinity, or, the actual infinity is not the infinity from the "infinity is unbounded" axiom for all infinity, while the infinity from the "infinity is unbounded" axiom stands for all infinity because of the consensus and guarantees and supports.

Nevertheless, since ∃∞∀∞ is proven because ∞ is not ∞, and for the use on Huo Jian Hua's Definition of Boundedness, everybody just has to honor Huo Jian Hua's guaranteed contractual rights-benefits up to infinity ∃∞∀∞ unbounded infinity to the end and live with ∃∞∀∞ because this infinity is not ∞ ∀∞, and vice versa, ∞ is not this infinity ∀∞, seriously. Thank Huo Jian Hua for His Definition of Boundedness.

「不存在」、「存在」、∃∞∀∞及∃∅⇔∄∅

此篇先舉例討論無限（以∞指稱）在處於不存在的狀況下。然而，若要證明∃∞∀∞（存在無限全量無限），我們需有〈「存在」的定義〉：∃x ⇔ x 即非∞∀x。（在此就認真地為了感謝霍建華而感謝霍建華。「非∞」即是「存在」的精隨，因為現在也成了是霍建華的論點了！）然而要證明〈「存在」的定義〉，我們需有〈容器的定義〉、〈元素的定義〉及霍建華〈有界的定義〉。 以此途徑可讓我們以後查對〈一切無限皆非無限〉定理。〈「存在」的定義〉及〈「不存在」的定義〉在一起可以證明∃∅⇔∄∅。我們也將在此篇中廣泛的應用來自上篇[覽 XY1]證明的[x 即 y]⇒[x 屬於 y]∀x、y。由於使用了霍建華〈有界的定義〉以證明〈一切無限皆非無限〉定理的原故，根據霍建華〈有界的定義〉使用契約保證，一切權力益全都歸霍建華。

為何需要證明∃∞∀∞？

那是因為數學在使用無限。根據以下〈「不存在」的定義〉，倘若∞不存在，則∞即∅：

∄x 若且唯若 x 即∅∀x。

證明〈「不存在」的定義〉：[∄x]⇔[x 即∅] ∀x。

證： 以∅指稱空無。

x	∄x	x 即∅	[∄x]⇔[x 即∅]
是	否	否	是
否	否	否	是
∅	是	是	是

[覽 NE]

x	∄x	x 即∅	[∄x]⇔[x 即∅]
∅	是	是	是

證明完畢。

所以，當無限被認為無限不存在時，其將可導致瑕積分、無限級數以及無限序列成為空無或是無定義。以下瑕積分解答 1∊ℝ 為例：

$$\int_1^\infty \mathrm{x}^{-2}\mathrm{dx} = -\mathrm{x}^{-1}|_1^\infty = \frac{-1}{\mathrm{x}}|_1^\infty = [\lim_{\mathrm{x}\to\infty}\frac{-1}{\mathrm{x}}] - \frac{-1}{1} = 0 + 1 = 1。$$

假設無限不存在，則以上的瑕積分就會成為以下這樣：

$$\int_1^\varnothing \mathrm{x}^{-2}\mathrm{dx} = -\mathrm{x}^{-1}|_1^\varnothing = \frac{-1}{\mathrm{x}}|_1^\varnothing = \frac{-1}{\varnothing} - \frac{-1}{1} = \frac{-1}{\varnothing} + 1。$$

由於簡純化的$\frac{-1}{\varnothing}+1$的實數在 ℝ 內是空無的，所以$\frac{-1}{\varnothing}+1$在 ℝ 內是空無的，而不是非空無的解答$\int_1^\infty \mathrm{x}^{-2}\mathrm{dx} = 1$在 ℝ 內。

舉例級數 S= $\sum_{n=0}^{\infty}\frac{1}{2^n} = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$，因此$\frac{S}{2} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$。在等號兩邊各加 1，

$1+\frac{S}{2} = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ ，因此$1+\frac{S}{2} = S$，所以$S=2$。倘若無限不存在，則$S=\sum_{n=0}^{\varnothing}\frac{1}{2^n}$。$\sum_{n=0}^{\varnothing}\frac{1}{2^n}$展開牽涉一項$(\frac{1}{2^0}+\frac{1}{2^\varnothing}) = (1+\frac{1}{2^\varnothing})$，因此 S ＝ $1+\frac{1}{2^\varnothing}+\cdots$。所以 S 即空無在 ℝ 內，而不是非空無的$S=\sum_{n=0}^{\infty}\frac{1}{2^n}=2$在 ℝ 內。

參考下列序列$a_{k+1}=a_k+a_{k-1}$，當$a_0=0$、$a_1=1$、$a_2=1$、$a_3=2$、$a_4=3$、$a_5=5$、$a_6=8$、…。所以$(\frac{a_{k+1}}{a_k})_{k=1}^{\infty}=\left(1,2,\frac{3}{2},\frac{5}{3},\frac{8}{5},\dots\right)$，$\lim_{k\to\infty}\frac{a_{k+1}}{a_k}=\frac{1+\sqrt{5}}{2}$。若無限不存在，則$(\frac{a_{k+1}}{a_k})_{k=1}^{\varnothing}$，$\lim_{k\to\varnothing}\frac{a_{k+1}}{a_k}=\frac{a_\varnothing}{a_\varnothing}\stackrel{?}{=}1$為無定義，而並非$\lim_{k\to\infty}\frac{a_{k+1}}{a_k}=\frac{1+\sqrt{5}}{2}$。

在以下再重述及對比前述舉例：

$$\left(\int_1^\infty \mathrm{x}^{-2}\mathrm{dx}=1\right)\ \textit{對照}\ \left(\int_1^\varnothing \mathrm{x}^{-2}\mathrm{dx}=\varnothing\right)$$

$$\left(S=\sum_{n=0}^{\infty}\frac{1}{2^n}=2\right)\ \textit{對照}\ \left(\sum_{n=0}^{\varnothing}\frac{1}{2^n}=\varnothing\right)$$

$$\left(\lim_{k\to\infty}\frac{a_{k+1}}{a_k}=\frac{1+\sqrt{5}}{2}\right)\ \textit{對照}\ \left(\lim_{k\to\varnothing}\frac{a_{k+1}}{a_k}=\frac{a_\varnothing}{a_\varnothing}\stackrel{?}{=}1\right)$$

這些$\int_1^\infty \mathrm{x}^{-2}\mathrm{dx}=1$，$\sum_{n=0}^{\infty}\frac{1}{2^n}=2$ *以及* $\lim_{k\to\infty}\frac{a_{k+1}}{a_k}=\frac{1+\sqrt{5}}{2}$展現出數學使用無限以求得不空無的解答，而不是把無限被認為是空無的（即不存在的）而導致於 ℝ 無定義的運算，或是在 ℝ 內的空無。所以，數學必須有無限存在的正式的證明。雖然根據〈「存在」的定義〉，[∞即非∞∀∞]已等效於辯解[∃∞∀∞]，以及，[∞即非∅∀∞]從之前證明已展現∞的存在並非微不足道，此篇更以數學證明提供〈「存在」的定義〉的起源。

容器的定義

C 即一個容器若且唯若 x 在 C 內 ∀x、C。

證明〈容器的定義〉：[C 即一個容器]⇔[x 在 C 內] ∀x、C。

證：

根據已知做為一個定義而言「C 即一個容器」即為「C 即（x 的）一個容器」∴

C	x	C 即一個容器	x 在 C 內	[C 即一個容器] ⇔ [x 在 C 內]
是	是	是	是	是
是	否	否	否	是
是	∅	是	是	是
否	是	否	否	是
否	否	是	是	是
否	∅	是	是	是
∅	是	否	否	是
∅	否	否	否	是
∅	∅	是	是	是

[覽 C]

在 x=是， 以及， y=是

C	x	C 即一個容器	x 在 C 內	[C 即一個容器] ⇔ [x 在 C 內]
是	是	是	是	是

[覽 C_1]

證明完畢。

眾容器

[是]、[否]、∅及 x 皆為容器。然而， 無限不是一個容器的情況下「無限即無界」公理才可支承。

證明 x 在 x 內 ∀x。

證明： 根據〈「空無」的支援證明〉[覽 EQ](3)，[x=x]⇒[x 在 x 內] ∀x。證明完畢。

證明[是]即一個容器∀[是]。

證： [是=是]⇒[是在是內]∴[是]即[是]的容器∴[是]即一個容器∀[是]。證明完畢。

證明[否]即一個容器∀[否]。

證： [否=否]⇒[否在否內]∴[否]即[否]的容器∴[否]即一個容器∀[否]。證明完畢。

證明∅即一個容器∀∅。

證： ∅在∅內∴∅即∅的容器∴∅即一個容器∀∅。證明完畢。

證明 x 即一個容器∀x。

證：[x=x]⇒[x 在 x 內] ∴x 即一容器∀x。

x	x 在 x 內	[C 即一個容器] ⇔ [x 在 C 內]	x 即一個容器
是	是	是	是
否	是	是	是
∅	是	是	是

[覽 C_x]

[x 在 x 內]排皆登錄為[是] ∴根據定義[[C 即一個容器] ⇔ [x 在 C 內]]排皆登錄為[是]，∴在[x 即一個容器]排皆被解答為[是]。∴ x 即一個容器∀x。證明完畢。

證明[x 即 y] ⇒ [y 即一個容器] ∀x、y。

證：

x	y	x 即 y	x 在 y 內	y 即（x 的）一個容器	[x 即 y] ⇒ [y 即一個容器]
是	是	是	是	是	是
是	否	否	否	否	是
是	∅	否	否	否	是
否	是	否	否	否	是
否	否	是	是	是	是
否	∅	否	否	否	是
∅	是	否	是	是	是
∅	否	否	是	是	是
∅	∅	是	是	是	是

[覽 C_2]

證明完畢。

證明無限即非一個容器∀無限。

證：　註：[x 即 y] ⇒ [x 屬於 y] ∀x、y。

　　假設無限即一個容器∴無限屬於一個容器∴無限即有界，根據霍建華〈有界的定義〉。但，無限即非有界，（無限即非有界）又（無限即有界）➔🡐∀無限∴無限即非一個容器∀無限。證明完畢。

存在的精隨：非∞。

　　我們已使用了霍建華〈有界的定義〉以證明∞即非一個容器∀∞。我們要使用〈容器的定義〉證明以下[是]、[否]、∅、x 或（非∞）皆非∞。以下諸（非∞）的證明是為了〈「存在」的定義〉真值表內登錄值所做的鋪設準備。

證明[是]即非無限∀[是]。

證：　假設[是]即無限 ∴無限即一個容器。但，無限即非一個容器，（無限即非一個容器）又（無限即一個容器）➔🡐 ∀[是]，∴[是]即非無限∀[是]。證明完畢。

證明[否]即非無限∀[否]。

證：　假設[否]即無限 ∴無限即一個容器。但，無限即非一個容器，（無限即非一個容器）又（無限即一個容器）➔🡐 ∀[否]，∴[否]即非無限∀[否]。證明完畢。

證明∅即非無限∀∅。

證：　假設∅即無限 ∴無限即一個容器。但，無限即非一個容器，（無限即非一個容器）又（無限即一個容器）➔🡐 ∀∅，∴∅即非無限∀∅。證明完畢。

證明非∞即非無限∀x。

證：　假設非∞即無限 ∴無限即一個容器。但，無限即非一個容器，（無限即非一個容器）又（無限即一個容器）➔🡐 ∀非∞，∴非∞即非無限∀非∞。證明完畢。

元素的定義

(元素 x≡x)∈C 若且唯若 x 在 C 內 ∀x、C。

證明〈元素的定義〉:[(元素 x≡x)∈C]⇔[x 在 C 內] ∀x、C。

證:

C	x	x∈C	x 在 C 內	[x∈C] ⇔ [x 在 C 內]
是	是	是	是	是
是	否	否	否	是
是	∅	是	是	是
否	是	否	否	是
否	否	是	是	是
否	∅	是	是	是
∅	是	否	否	是
∅	否	否	否	是
∅	∅	是	是	是

[覽 EL]

在 x=是, 以及, y=是

C	x	[x∈C]	x 在 C 內	[x∈C] ⇔ [x 在 C 內]
是	是	是	是	是

[覽 EL_1]

證明完畢。

諸元素

[是]、[否]、∅及 x 皆為元素。然而，無限不是一個元素的情況下「無限即無界」公理才可支承。

證明[是]即一個元素∀[是]。

證: [是]即[是]的一個容器∀[是] ∴[是]在[是]內∴元素[是]≡[是]∈[是] ∴[是]即一個元素∀[是]。證明完畢。

證明[否]即一個元素∀[否]。

證: [否]即[否]的一個容器∀[否] ∴[否]在[否]內∴元素[否]≡[否]∈[否] ∴[否]即一個元素∀[否]。證明完畢。

證明∅即一個元素∀∅。

證: ∅已經在∅內∴元素∅≡∅∈∅ ∴∅即一個元素∀∅。證明完畢。

證明 x∈x ∀x。

證:

x	x 在 x 內	[x∈x] ⇔ [x 在 x 內]	x∈x
是	是	是	是
否	是	是	是
∅	是	是	是

[Table EL_x]

[x 在 x 內]排皆登錄為[是] ∴根據定義[[x∈x] ⇔ [x 在 x 內]]排登錄皆為[是]，∴ 在[x∈x]排皆被解答為[是]。∴ x∈x ∀x。證明完畢

證明無限即非一個元素∀無限。

證: 假設無限即一個元素 ∴無限屬於一個元素∴無限即有界，根據霍建華〈有界的定義〉。但，無限即非有界，(無限即非有界) 又 (無限即有界) ➔🡸∀無限∴無限即非一個元素∀無限。證明完畢。

證明[x∈C] ⇔ [x 屬於 C] ∀x、C。

證： 根據定義所有[x 在 C 內]⇔[x∈C]皆被錄為[是]：

x	C	[x 在 C 內]⇔[x∈C]	x 在 C 內	x∈C	x 屬於 C	[x∈C]⇔[x 屬於 C]
是	是	是	是	是	是	是
是	否	是	否	否	否	是
是	∅	是	否	否	否	是
否	是	是	否	否	否	是
否	否	是	是	是	是	是
否	∅	是	否	否	否	是
∅	是	是	是	是	是	是
∅	否	是	是	是	是	是
∅	∅	是	是	是	是	是

[覽 C_3]

證明完畢。

「存在」的定義

∃x 若且唯若 x 即非無限 ∀x。

（註：「∃」符號表示有關存在的量詞。「∀」符號表示全體的量詞。）

證明〈「存在」的定義〉：[∃x]⇔[x 即非無限] ∀x。

證：

行＼排	1	2	3	4
1	x	∃x	x 即非無限	[∃x]⇔[x 即非無限]
2	是	是	是	是
3	否	是	是	是
4	∅	是	是	是
5	非無限	是	是	是
6	無限	是	是	是

[覽 E1]

在之前已證明 [是]、[否]或空無各自以元素的方式存在，因其存在，所以這些元素被列在[覽 E1]的第一排，所以（行 2，排 2）、（行 3，排 2）及（行 4，排 2）各自被登錄為[是]。 [是]、[否]及∅皆非無限，所以（行 2，排 3）、（行 3，排 3）及（行 4，排 3）各自被登錄為[是] 。（行 5，排 2）登錄為[是] ，由於每一個從[覽 E1]存在的 [是]、[否]或∅皆非無限。非無限即非無限致使（行 5，排 3）為[是]。無限在「無限即無界」公理內，因此根據定義無限即「無限即無界」公理內的一個元素，所以，無限即一個元素。因為在「無限即無界」公理內的無限(以∞指稱)是一個元素，一個也可以在[覽 E1]（行 6，排 1）內的一個元素，如此的無限即非無限，下段將給予展示，以至於此〈「存在」的定義〉證明內，我們可進一步檢查該（行 6，排 1）的登錄的可行性。

[∃無限] ⇒ [無限即非無限]：

根據「無限即無界」公理的共識、保證及支持，「無限即無界」公理支承下<u>∀無限</u>，有存在無限在「無限即無界」的公理內∀無限、因此（行 6，排 2）被註為[是]。然而，這樣從「無限即無界」公理來的無限並非無

限。在「無限即無界」內的無限即一個元素。但，無限即非一個元素，由於（無限即非一個元素）又（無限即一個元素），所以無限作為一個元素在「無限即無界」內即空無。然而∅即非無限∴該無限做為一個∅的佔位符號在「無限即無界」即非無限∴［∃無限］⇒［無限即非無限］。

［無限即非無限］⇒［∃無限］：

無論[無限即非無限]即[是]或[否]，根據「無限即無界」公理的共識、保證及支持，當「無限即無界」公理支承下，有存在無限在「無限即無界」的公理內<u>∀無限</u>∴［無限即非無限］⇒［∃無限］。

∴[∃無限]⇔[無限即非無限]∴在[覽 E1]（行 6，排 4）被註為[是]。由於（行 6，排 2）為[是]所以（行 6，排 3）必然為[是]才能使得（行 6，排 4）能被[是]支承著。

在第 2 排內的每一行登錄皆為[是]及在第 3 排的每一行登錄為[是]，∴[覽 E1]結論在第 4 排每一行皆被登錄為[是] ∀x ∴[∃x]⇔[x 即非無限] ∀x。證明完畢。

證明∃∞∀∞。（證明存在無限全量無限）

證：

前篇中已證明∞即非∞∀∞，∴∞即非∞∴∃∞。

重述〈「存在」的定義〉：[∃ x]⇔[x 即非∞]∀x。∴[x 即非∞]⇔[∃ x]∀x。

∴[∃∞]⇔[∞即非∞]∀∞ ∴[∞即非∞]⇔[∃∞]∀∞ ∴∃∞∀∞。證明完畢。

查對∃∞∀∞。

證：

[覽 E1]的（行 6，排 3）必然為[是] ∴∞即非∞∴∃∞。

重述〈「存在」的定義〉：[∃ x]⇔[x 即非∞]∀x。∴[x 即非∞]⇔[∃ x]∀x。

∴[∃∞]⇔[∞即非∞]∀∞ ∴[∞即非∞]⇔[∃∞]∀∞ ∴∃∞∀∞。證明完畢。

前篇已證明無限即非∅，同時此篇也證明∃∞∀∞，此所以此無限的存在並非是微不足道，經由霍建華〈有界的定義〉使用契約保證，一切∞(無限)權力益全都歸霍建華，因為濟公保證要無限不是無限是認真的。

查對∞即非∅ ∀∞。

證： 假設∞即∅，「無限即無界」公理則成為「∅即無界」∴無界即一容器，∴無界屬於一容器 ∴無界即有界，根據霍建華〈有界的定義〉。但，無界即非有界，(無界即非有界)又(無界即有界) ➔⟵ ∀∞ ∴∞即非∅ ∀∞。證明完畢。

查對∞即非∞∀∞。

證： ∵∃∞∀∞∴∃∞ ∴[∃∞]⇔[∞即非∞]∀∞ ∴∞即非∞∀∞。證明完畢。

證明∃無界∀無界。（證明存在無界全量無界）

證： 假設無界即∞ ∴∞即一個容器。但，∞即非一個容器，（∞即非一個容器）又（∞即一個容器）➔⟵ ∀無界 ∴無界即非∞∀無界∴∃無界∀無界。證明完畢。

查對∃無界∀無界。

證： 來自[覽 B1]於〈證明無限不是無限〉篇內，以及根據「無限即無界」公理的共識、保證及支持，「無限即無界」公理支承下<u>∀無界</u>，有存在無界在「無限即無界」的公理內∀無界。∴∃無界∀無界。證明完畢。

所以，一切全部無界的權力益全都歸霍建華∀無界，根據對霍建華的〈有界的定義〉使用契約保證。

證明無界即非∅∀無界。

證：假設無界即∅ ∴無界屬於∅ ∴無界即有界。但，無界即非有界，（無界即非有界）又（無界即有界），➔⟵ ∀無界∴無界即非∅∀無界。證明完畢。

查對無界即非∅ ∀無界。

證：假設無界即∅ ∴「無限即無界」公理則成為「無限即∅」，所以∞即∅。但，∞即非∅，(∞即非∅)又(∞即∅) ➔🡐 ∀無界 ∴無界即非∅ ∀無界。證明完畢。

所以，無界的存在並非微不足道，因為無界即非∅。根據〈有界的定義〉使用契約保證一切無界權力益全都歸霍建華，以及，因為無界不是無限全量無界。

證明無界即非無限 ∀無界。

證： 假設無界即無限 ∴無界屬於無限 ∴無界即有界，根據霍建華〈有界的定義〉。但，無界即非有界，（無界即非有界）又（無界即有界）➔🡐 ∀無界 ∴無界即非無限∀無界。證明完畢。

查對無界即非無限 ∀無界。

證： ∃無界∀無界，來自之前的證明 ∴無界即非無限，據定義。證明完畢。

所以，根據霍建華〈有界的定義〉使用契約保證，一切無界的權力益也全都歸霍建華，因為濟公保證要證明無限不是無限——是認真的。

證明 x 即非∞∀x。

證明： 從[覽 E1]真值表中所有在第三排[x 即非∞]之下皆為[是]，一一對在每一個在第一排全部的 x ∴x 即非∞ ∀x。 證明完畢。

查對 x 即非∞∀x。

證： 假設 x 即∞ ∴∞即一個容器。但，∞即非一個容器，（∞即非一個容器）又（∞即一個容器）➔🡐 ∀x，∴x 即非∞∀x。證明完畢。

證明∃x∀x。

證： x 即非∞ ∀x ∴x 即非∞ ∴∃x∀x。證明完畢。

查對∃x∀x。

證：

從[覽 E1]真值表中所有在第二排[∃x]之下皆為[是]，一一對在每一個在第一排全部的 x ∴∃x∀x。證明完畢。

值得注意的是 x 可以微不足道的存在由於 x 可為∅，然而根據〈「不存在」的定義〉x 又不存在。有鑑於此狀況本篇將會證明∃∅⟺∄∅∀∅。

證明∃…等等 ∀…等等。

證：

假設 …等等即∞∴∞即一個容器。但，∞即非一個容器，（∞即非一個容器）又（∞即一個容器）➔🡐 ∀…等等 ∴ …等等即非∞ ∴∃…等等 ∀…等等。證明完畢。

「∃…等等 ∀…等等」。全部「…等等」權力益都歸霍建華，根據霍建華〈有界的定義〉使用契約保證，因為「…等等」即非無，限全量「…等等」。

在〈「不存在」的定義〉內的無限

此篇文章應用〈不存在定義〉以否定「無限不存在」。將來自〈「存在」的定義〉證明的[覽 E1]與來自〈「不存在」的定義〉證明的[覽 NE]合併至以下[覽 E_N]，在填好來自[覽 NE]所空缺的灰色空格：

行＼排	1	2	3	4	5	6	7	
1	x	∃x	x 即非∞	[∃x]⇔[x 即非∞]	∄x	x 即∅	[∄x]⇔[x 即∅]	
2	是	是	是	是	否	否	是	
3	否	是	是	是	否	否	是	
4	∅	是	是	是	是	是	是	
5	非∞	是	是	是	否	否	是	
6	∞	是	是	是	否	否	是	[覽 E_N]

（行 5，排 5）的陳述為：不存在非無限∀非無限。反例：[是]即非無限並且根據〈「存在」的定義〉存在[是]，所以（行 5，排 5）被登錄為［否]。

（行 5，排 6）的陳述為：非無限即空無∀非無限。反例：無限即非無限並且無限即非∅，所以（行 5，排 6）被登錄為[否]。

（行 6，排 5）的陳述為：不存在無限∀無限。此陳述是被[否]定的，由於∃∞∀∞已被證明。

（行 6，排 6）的陳述為：無限即空無∀無限。此陳述是被[否]定的，由於無限即非∅已被證明，否則「無限即無界」公理不能支承。

（行 5，排 7）與（行 6，排 7）皆被登錄為[是]，因來自第 5 排及第 6 排的綜合結果。因為全部的在第 7 排的數值皆為[是]，所以在[∄x]⇔[x 即∅]∀x 依然是確立的同時[非∞]及[∞]行也皆在[覽 E_N]內。

第 6 行全都是在闡述無限，所以當 x≡∞，[∄x]⇔[x 即∅]則成為[∄∞]⇔[∞即∅]。然而根據[覽 E_N]，[∄∞]已被登錄為[否]，其已否定無限的不存在因為[∞即∅]為[否]定的且由於∞即非∅，否則「無限即無界」公理將不支承。因此所以∄∞被登錄為[否]，其否定「無限不存在」的主張，根據共識、保證及支持的「無限即無界」公理也可以同時支承著。多謝有霍建華的〈有界的定義〉。

∃∅⇔∄∅

∄∅∀∅及∃∅∀∅皆為正當的陳述，∄∅∀∅是根據〈「不存在」的定義〉以及∃∅∀∅是根據〈「存在」的定義〉。舉例以 x 表示 π∈ℕ。但，（π∈ℕ）即空無∴x 即∅ ⇒ ∄x ∴∄（π∈ℕ）

∴∄（π∈ℕ）⇔（π∈ℕ）即∅∀（π∈ℕ），

∴∄（ ∅ ）⇔（ ∅ ）即∅∀（ ∅ ）。

∴∄∅ ⇔ ∅即∅∀∅。∅既已空無，因此∄∅。以下證明∄∅∀∅。

證明∄∅∀∅。

證： 重述〈「不存在」的定義〉：[∄x]⇔[x 即∅]∀x。

∅即∅ ∴ ∄∅∀∅根據定義。證明完畢。

然而，∅存在由於∅即非∞雖然∅的存在是微不足道的。以下證明∃∅∀∅。

證明$\exists\varnothing\forall\varnothing$。

證：　重述〈「存在」的定義〉：$[\exists x]\Leftrightarrow[x\text{ 即非}\infty]\forall x$。

以 C 指稱為一個容器。假設$\varnothing$即∞ $\therefore$ ∞即 C。但，∞即非 C，（∞即非 C）又（∞即 C）$\rightarrow\leftarrow$ $\forall\varnothing$ $\therefore$ $\varnothing$即非∞ $\therefore$ $\exists\varnothing\forall\varnothing$。證明完畢。

證明$\exists\varnothing\Leftrightarrow\nexists\varnothing\ \forall\varnothing$。

證：　[覽 E_N]的第 4 行專注在$\varnothing$，因此該第 4 行為全量$\varnothing$。
（行 4，排 2）被登錄為[是]且陳述著「存在$\varnothing$」，也就是$\exists\varnothing$；
（行 4，排 5）被登錄為[是]且陳述著「不存在$\varnothing$」，也就是$\nexists\varnothing$。
$\exists\varnothing$即[是]與$\nexists\varnothing$即[是]皆同時出自同一個第 4 行 $\therefore\exists\varnothing\Leftrightarrow\nexists\varnothing\ \forall\varnothing$。證明完畢。

$\varnothing$的存在是微不足道的其可成為不存在：$\exists\varnothing\Leftrightarrow\nexists\varnothing\forall\varnothing$。但是有關於$\infty$，若$\infty$即$\varnothing$，則$\infty$的存在將會是微不足道的。然而，因為在前篇已經證明$\infty$即非$\varnothing$，並且還進一步的證明了$\exists\infty\forall\infty$，所以可以對$\infty$的存在有「非微不足道」的結論。這也可以解釋為什麼當無限被假定為空無以致無限微不足道存在，其後與無限並非微不足道存在結論造成矛盾，瑕積分、無限級數以及無限序列會導致成為荒謬地空無或是無定義。感謝霍建華的〈有界的定義〉，一切權力益全都歸霍建華，根據霍建華〈有界的定義〉使用契約的保證。

證明$\exists(\pi\in\mathbb{N})\Leftrightarrow\nexists(\pi\in\mathbb{N})\ \forall(\pi\in\mathbb{N})$。

證明：

(1) 假設 $\pi\in\mathbb{N}$ 即非$\varnothing$$\therefore$ π 在 $\mathbb{N}$ 內。但，π 不在 $\mathbb{N}$ 內，（π 不在 $\mathbb{N}$ 內）又（π 在 $\mathbb{N}$ 內）$\rightarrow\leftarrow$ $\forall\pi\in\mathbb{N}$ $\therefore\pi\in\mathbb{N}$ 即非$\varnothing$ $\therefore\nexists(\pi\in\mathbb{N})$。

(2) 假設（$\pi\in\mathbb{N}$）即∞ $\therefore\infty$即一個容器 $\therefore\infty$屬於一個容器 $\therefore\infty$即有界，根據霍建華〈有界的定義〉。但，無限即非有界，（無限即非有界）又（無限即有界）$\rightarrow\leftarrow$ $\forall(\pi\in\mathbb{N})$ $\therefore(\pi\in\mathbb{N})$即非$\infty$ $\therefore\exists(\pi\in\mathbb{N})$。

$\exists(\pi\in\mathbb{N})\Leftrightarrow\exists\varnothing;\ \nexists\varnothing\Leftrightarrow\nexists(\pi\in\mathbb{N})\ \therefore$

$\exists(\pi\in\mathbb{N})\Leftrightarrow\exists\varnothing\Leftrightarrow\nexists\varnothing\Leftrightarrow\nexists(\pi\in\mathbb{N})$

$\therefore\ \exists(\pi\in\mathbb{N})\Leftrightarrow\nexists(\pi\in\mathbb{N})\ \forall(\pi\in\mathbb{N})$。證明完畢。

$\pi\in\mathbb{N}$ 的存在是微不足道的其可成為不存在，就如同$[\exists(\pi\in\mathbb{N})]\Leftrightarrow[\nexists(\pi\in\mathbb{N})]$所做的陳述。

在數學裡有時會出現無限為一個題目的解答，然而數學老師會說這樣的解答是「不存在」或「無限不是一個數目」（無限根本都不是無限它自己）。數學題目經常是要在一個數目集合（例如實數集合 $\mathbb{R}$）裡面找出答案。無無限並非在實數集合內，因此答案在 $\mathbb{R}$ 內是空無的，所以答案不存在於 $\mathbb{R}$ 內，也就是說$\nexists\infty\in\mathbb{R}\forall\infty\in\mathbb{R}$。因為$\infty\in\mathbb{R}$即$\varnothing$，所以以下證明應用了$\exists\varnothing\Leftrightarrow\nexists\varnothing\forall\varnothing$。

證明$\exists(\infty\in\mathbb{R})\Leftrightarrow\nexists(\infty\in\mathbb{R})\ \forall(\infty\in\mathbb{R})$。

證：　假設$\infty\in\mathbb{R}$ 即非$\varnothing$$\therefore\infty$必然是在 $\mathbb{R}$ 內的一個非空無元素$\therefore(\infty\in\mathbb{R})\Rightarrow(\infty$屬於 $\mathbb{R})$ $\therefore\infty$即有界，根據霍建華〈有界的定義〉。但，∞即非有界，（無限即非有界）又（無限即有界）$\rightarrow\leftarrow$ $\forall\infty\in\mathbb{R}$ $\therefore$ $\infty\in\mathbb{R}$ 即$\varnothing$ $\therefore\exists(\infty\in\mathbb{R})\Leftrightarrow\nexists(\infty\in\mathbb{R})\ \forall(\infty\in\mathbb{R})$。證明玩畢。

查對$\nexists\infty\in\mathbb{R}\ \forall\infty\in\mathbb{R}$，重述〈「不存在」的定義〉：

$$[\nexists x]\Leftrightarrow[x\text{ 即}\varnothing]\forall x\text{，或是，}[x\text{ 即}\varnothing]\Leftrightarrow[\nexists x]\forall x\text{。}$$

以 x 指稱$(\infty\in\mathbb{R})$。但，$(\infty\in\mathbb{R})$即空無 $\therefore$ x 即$\varnothing$ $\therefore$ $\nexists x\ \forall x$ $\therefore\nexists\infty\in\mathbb{R}\ \forall\infty\in\mathbb{R}$。

以下證明查對$\exists\infty\in\mathbb{R}\ \forall\infty\in\mathbb{R}$。

證明∃∞∈ℝ ∀∞∈ℝ。

證： 假設∞∈ℝ 即∞∴∞即一個容器。但，∞即非一個容器，（∞即非一個容器）又（∞即一個容器）➔⃪ ∀∞ ∴ ∞∈ℝ 即非∞ ∴ ∃∞∈ℝ ∀∞∈ℝ 根據〈「存在」的定義〉。證明完畢。

∞∈ℝ 其實是∅的佔位符號。∞∈ℝ 的存在是微不足道的其可成為不存在，就如同[∃(∞∈ℝ)]⇔[∄(∞∈ℝ)]內所做的陳述。

無限即非一個數目。

證明無限即非一個數目∀無限。

證： 假設無限即一個數目∴無限屬於一個數目∴無限即有界，根據霍建華〈有界的定義〉。但，無限即非有界，（無限即非有界）又（無限即有界）➔⃪ ∀無限∴無限即非一個數目∀無限。證明完畢。

證明無限即非 y∀無限。

證： 假設無限即 y∴無限屬於 y∴無限即有界，根據霍建華〈有界的定義〉。但，無限即非有界，（無限即非有界）又（無限即有界）➔⃪ ∀無限∴無限即非 y∀無限。證明完畢。

證明無限即非 y∀y。

證： 假設無限即 y∴無限屬於 y∴無限即有界，根據霍建華〈有界的定義〉。但，無限即非有界，（無限即非有界）又（無限即有界）➔⃪ ∀y∴無限即非 y∀y。證明完畢。

證明無限即非 y∀無限、y。

證： 假設無限即 y∴無限屬於 y∴無限即有界，根據霍建華〈有界的定義〉。但，無限即非有界，（無限即非有界）又（無限即有界）➔⃪ ∀無限、y ∴無限即非 y∀無限、y。證明完畢。

從總體上來說，無限不存在於任何容器 C 內，也就是∄∞∈C ∀∞∈C。∞∈C 的存在是微不足道的其可成為不存在就如同以下[∃(∞∈C)]⇔[∄(∞∈C)]證明所做的陳述。

證明∃(∞∈C)⇔∄(∞∈C) ∀(∞∈C)。

證： 假設∞∈C 即非 ∴∞必然是在 C 內的一個非空無的元素 ∴(∞∈C) ⇒ (∞屬於 C) 根據[覽 C_3]∴∞即有界，根據霍建華〈有界的定義〉。但，∞即非有界，（∞即非有界）又（∞即有界）➔⃪ ∀∞∈C ∴ ∞∈C 即∅ ∴∃(∞∈C)⇔∄(∞∈C) ∀(∞∈C)。證明完畢。

查對∃∞∈C ∀∞∈C。

證： 假設∞∈C 即∞ ∴∞即一個容器。但，∞即非一個容器，（∞即非一個容器）又（∞即一個容器）➔⃪ ∀∞ ∴ ∞∈C 即非∞∴ ∃∞∈C ∀∞∈C 根據〈「存在」的定義〉。證明完畢。

∞∈C 是∅的一個佔位符號由於∄∞∈C。

查對∄∞∈C ∀∞∈C。

證： 假設∃∞∈C ∴∞ 屬於 C 根據[覽 C_3] ∴∞即有界，根據霍建華〈有界的定義〉。但，∞即非有界，（∞即非有界）又（∞即有界）➔⃪ ∀∞∈C ∴ ∄∞∈C ∀∞∈C。證明完畢。

在這一篇內我們證明了∃∞∀∞，查對了〈一切無限皆非無限〉定理以及證明了∄∅⇔∃∅連帶著使用了霍建華〈有界的定義〉，〈「不存在」的定義〉以及〈「存在」的定義〉。經有使用了霍建華〈有界的定義〉，我們可以證明無限即非一個數目。∃(∞∈C)⇔∄(∞∈C) ∀(∞∈C)的結論說明無限不存在於任何一個容器 C 內（例如這個宇宙就是一個容器）。這樣的無限是剩下的在任何的研究論文之外因為研究論文就可以成為一個容器。然而在[覽 E_N]內的〈「不存在」定義〉依然否定「無限不存在」的主張，而同時∃∞∀∞提倡無限的存在，根據來自共

識、保證即支持的「無限即無界」公理。為什麼？在∃∞∀∞內的無限可以存在是基於在「無限即無界」公理內寫下的無限，然而在∃∞∀∞內的無限並非那無法可在任何論文內出現的無限。

「無限在不在這個宇宙內？」這樣的問題就如同在問：「***π***∈ℝ 在不在自然集合的宇宙內？」答案是否定的。無論你如何把***π***呈獻為自然數的世界其將會成為空無或不是***π***它自己，所以在自然數的宇宙內***π***會呈現得成為不存在。類似地，在這個宇宙內所呈獻的無限不是無限它自己，其而呈現為空無的，所以無限在這個宇宙內呈現為不存在。我們可以對「∞不是∞∀∞」做一個更簡單的解釋：來自「無限即無界」公理的無限不是實際上的無限，或是說，實際上的無限不是來自「無限即無界」公理的無限全量無限，而來自「無限即無界」公理的無限卻代表了全量無限由於共識、保證即支持的緣故。

無論如何，既然∃∞∀∞已被證明由於∞不是∞，而且又使用了霍建華〈有界的定義〉，大家就認帳兌現霍建華的權力益無限∃∞∀∞無界無限到底，與∃∞∀∞在一起生活著，因為這無限不是∞ ∀∞，反之亦然，∞不是這無限 ∀∞，這是認真的。感謝霍建華的〈有界的定義〉。